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Pressuremeter tests in sands

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The Paper is concerned with the use of a self-boring pressuremeter to obtain in situ measurements of the properties of sands. A brief description of the instrument and of its method of operation is given. A review of the interpretation of pressuremeter tests in sand is presented and a new interpretation is described in which allowance is made for the volume change that occurs in sand when it is sheared. It is assumed that the stress–dilatancy theory is valid, and that sand fails with a constant ratio of principal stresses. The analysis allows a value for the angle of internal friction ϕ' and a value for the angle of dilatation ν to be determined. Experimental results of tests in sand at the Wash and in the hydraulic fill at the Kernick dam are presented; the resulting values of ϕ' are compared with those deduced from cone penetration tests and triaxial tests. Values of shear modulus G and in situ horizontal stress σ'_h are also measured and reported.

L'étude a pour l'objet l'utilisation du pressiomètre auto-foreur afin d'obtenir des mesures in situ des propriétés de sables. Une courte description de l'instrument et de son mode d'emploi est donnée. Une analyse concernant l'interprétation des essais pressiométriques dans le sable est présentée, et une nouvelle interprétation est décrite dans laquelle est pris en compte le changement de volume qui se produit dans le sable après cisaillement. Il est montré que la théorie 'contrainte–dilatance' est juste et que lorsque le sable se rompt le rapport entre les contraintes principales reste constant. L'analyse permet la détermination des valeurs pour l'angle de frottement interne ϕ' et du coefficient de Poisson ν. Les résultats d'essais effectués dans le sable près du Wash et dans le remblai hydraulique du barrage de Kernick sont présentés. Les valeurs de ϕ' obtenues de cette façon sont comparées avec celles déduites à partir des essias de pénétration au cône et d'essais triaxiaux. On a en plus mesuré et présenté des valeurs pour le module de cisaillement G et pour la contrainte horizontale.

It is particularly difficult to sample sands and virtually impossible to obtain an undisturbed sample. Consequently, in situ tests are the most common method of determining the engineering properties of sands. Small-scale plate load tests, pressuremeter tests and penetration tests form the bulk of these in situ test methods. Owing to their simplicity the dynamic standard penetration test (SPT) and quasi-static Dutch cone test are used almost to the exclusion of other more expensive tests.

The determination of the basic properties of the soil from either the cone resistance, or the blow count N of the SPT is very subjective and depends on many factors such as the density and stress history of the soil, the in situ stresses, and the position of the water table. Both tests deform the soil to large strains and thus the behaviour of the sand at small strains is difficult to assess. In spite of these difficulties many attempts have been made to produce acceptable correlations to meet the needs of practising engineers.

Terzaghi and Peck (1948) established a correlation between SPT results and settlement of footings. Recently various modifications and refinements have been proposed in order to give better agreement between predicted and observed settlements. Bratchell *et al.* (1974) have pointed out that for a particular example the predictions can very by a factor of up to 14.

Gibbs and Holtz (1957) proposed a correlation between SPT results and the relative density of a sand deposit. However, an error analysis by Tavenas and La Rochelle (1972) shows that

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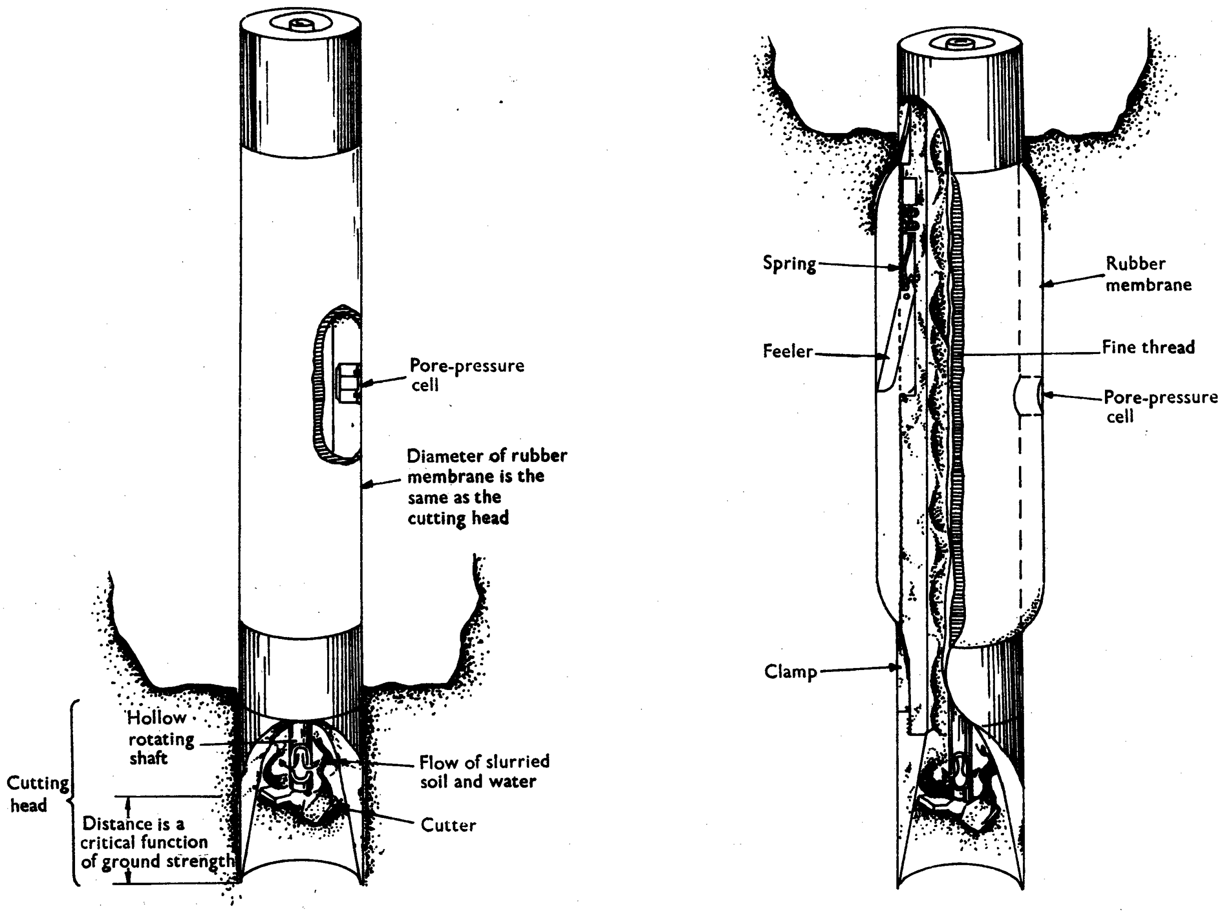

Fig. 1. Schematic diagram of self-boring pressuremeter

such a procedure cannot produce values of relative density with any degree of confidence and that such values are merely an educated guess. An example of the typically large scatter in correlations between the angle of shearing resistance, ϕ', and the resistance of Dutch cone penetrometers is given later in this Paper.

The above examples illustrate some of the problems of using such empirical correlations with the data of standard penetration or Dutch cone tests. It is suggested that this is because the SPT blow count or cone resistance is affected by many factors none of which can be taken fully into account when producing such a correlation. The more expensive plate loading tests or pressuremeter tests, however, do produce data that can be analysed more rigorously to produce engineering parameters directly.

A new method of determining the angle of internal friction ϕ' and the angle of dilatation ν from pressuremeter test data is presented in this Paper. In adopting a more rigorous approach, the Authors consider that the resulting in situ values of ϕ' and ν will be less sensitive to factors such as stress history and will be more truly representative of the engineering properties of the soil. The major advantage is that more confidence can be placed on the test results because factors such as overburden pressure, pore-water pressure and stress history are correctly accounted for in the test analysis. However, it should be recognized that these values of ϕ' are from plane strain tests and, consequently, they may be larger than those obtained from conventional triaxial tests.

The current programme of research at the University of Cambridge on in situ testing is concentrated on instruments that are inserted into the ground by means of a self-boring method. As a consequence, the pressuremeter is introduced into the soil so that there is

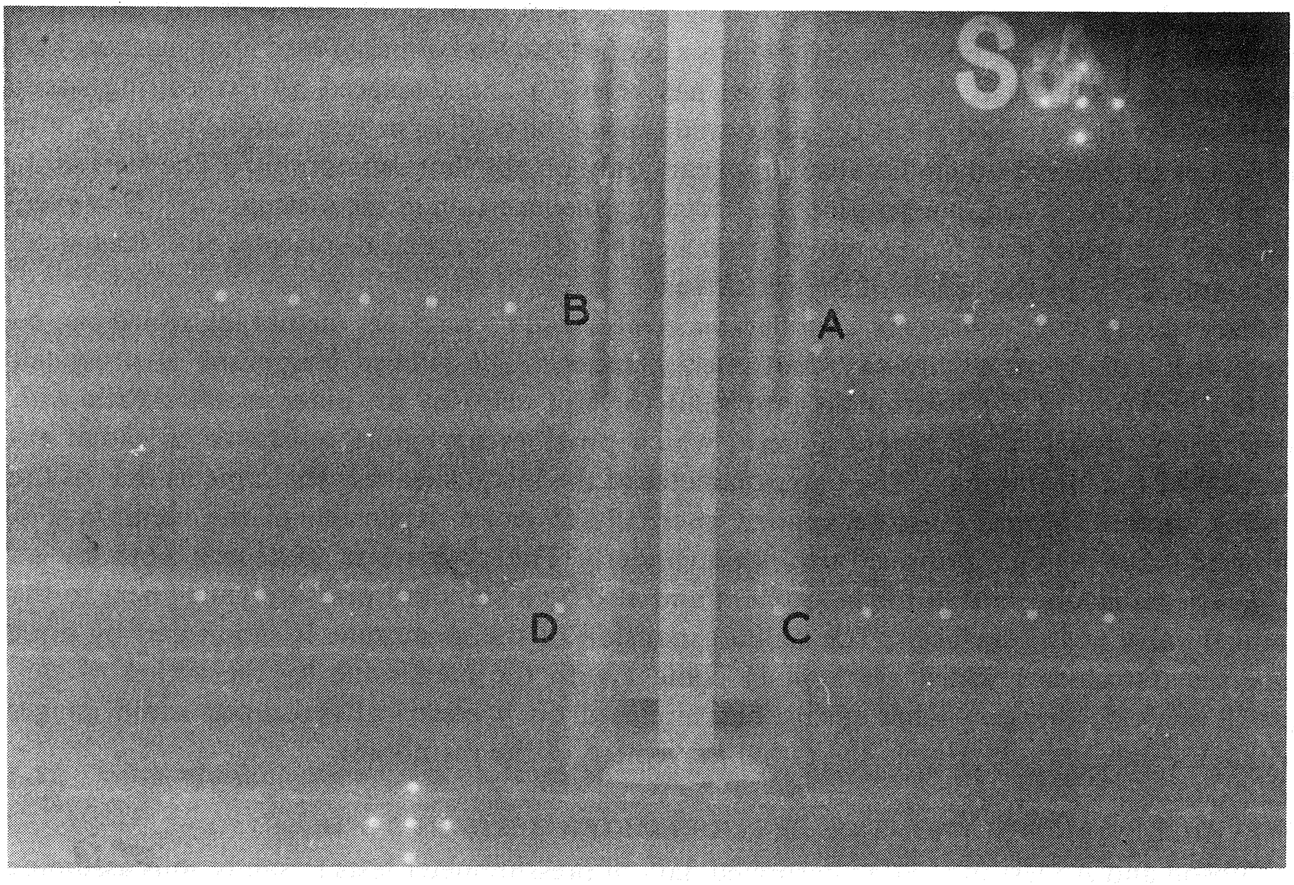

Fig. 2. Double exposure radiograph of sample of sand taken before and after insertion of self-boring instrument

minimal disturbance. The behaviour of the soil at both small and large values of strains may be observed accurately.

In the next section a brief description of the self-boring pressuremeter is given. A new analysis of the data is then presented, which takes account of the dilatation that occurs in medium dense sand as it fails. The analysis is applied to data of tests on sand, and also to a hydraulic fill at Kernick in Cornwall where a direct comparison is made with the results of Dutch cone tests.

THE SELF-BORING PRESSUREMETER

The overriding design requirement for the self-boring pressuremeter is that it should be capable of being introduced into the ground with the very minimum of disturbance. The method of achieving this for the pressuremeter developed at the University of Cambridge is illustrated in Fig. 1 showing a schematic section of the instrument. Very similar devices have been developed independently at the Laboratoires des Ponts et Chausées in France by Baguelin and Jézéquel (1975) and by Baguelin *et al.* (1974).

The basic instrument consists of a cylinder that is jacked steadily into the ground while the soil that enters the open bottom end of the cylinder is transported to the surface. This soil is removed by a rotating cutter and is flushed to the surface by water which is pumped down the inside of the cutter driving rods and up the annular space between the rods and the inside of the cylinder. The degree of disturbance caused by the insertion and operation of the instrument has been observed by Hughes (1973) under carefully controlled laboratory conditions by means of the radiographic technique developed by Arthur *et al.* (1964).

Figure 2 is the superposition of two radiographs taken before and after a prototype instrument was drilled into a bin of sand in which a central, vertical plane of lead shot markers had been placed. One lead shot, labelled A, has been hit and displaced by the cutting shoe; another lead shot labelled B has been removed during the drilling process and its image is only half as intense as the others because it appears only on the first radiograph. The lead shot, C, was out of plane and in a position unaffected by the insertion of the instrument. However, the other lead shot have hardly moved, and significantly the radial displacement of lead shot, D, is only 0·5% of the radius of the instrument.

The pressuremeter consists of an inflatable cell formed by a rubber membrane which can be expanded radially in a carefully controlled manner. The rubber membrane is protected during insertion of the instrument into sand (or stiff clay) by an outer metal sheath with longitudinal cuts like a Chinese lantern, which provides negligible resistance to radial expansion of the membrane. The outer diameter of the metal sheath is the same as that of the cutting shoe. The gas pressure applied internally to cause expansion of the membrane is supplied from a high pressure nitrogen cylinder. The pressure is measured both by an electrical transducer within the instrument and also by a standard pressure gauge next to the control valve at the surface. The radial expansion of the membrane is monitored by three separate transducers which are kept in contact with the membrane by the action of thin cantilever leaf springs. Each spring has mounted on it electrical resistance strain gauges which indicate very accurately the radial movement of the membrane.

PREVIOUS INTERPRETATIONS OF PRESSUREMETER TESTS IN SAND

Gibson and Anderson (1961) interpreted data of pressuremeter tests in sandy soils assuming that the sand behaved perfectly elastically until failure was reached, and that thereafter the sand continued to fail at constant ratio of effective stresses (as the stress level increased) and at constant volume. Their analysis related the applied effective pressure σ'_{r_a} with the volumetric strain of the pressuremeter $\Delta V/V$ (in terms of the *current* volume V) by the following expression:

$$\sigma'_{r_a} = \left(\frac{2\sigma'_{r_0}}{1+N}\right)\left[\left(\frac{E'}{2\sigma'_{r_0}(1+\mu')}\right)\left(\frac{1+N}{1-N}\right)\left(\frac{\Delta V}{V}\right)\right]^{(1-N)/2} \quad . \quad . \quad . \quad . \quad (1)$$

where σ'_{r_0} is the initial effective lateral stress, E' is Young's modulus, μ' the Poisson's ratio[1] and $N=(1-\sin\phi')/(1+\sin\phi')$ is the[2] ratio of the minor to major principal effective stresses at failure. Hence the value of N can be determined by plotting $\log \sigma'_r$ against $\log (\Delta V/V)$ and measuring the gradient of the resulting straight line which is equal to $\frac{1}{2}(1-N)$ or $\sin\phi'/(1+\sin\phi')$. The major flaw in this analysis is the assumption that, at failure, the sand deforms at constant volume.

Ladanyi (1963) adapted this analysis by making a more realistic assumption about the volume changes that occur in sand prior to failure. He idealized the relationship between the volumetric strain v and the engineering shear strain γ for granular materials as shown by the plots of Fig. 3, for initially dense and loose specimens. Failure was assumed to occur at constant effective stress ratio and at constant volume, at the stage of the test corresponding to point B.

Since conventional pressuremeter tests are taken to relatively high strains, Ladanyi considered only the failure portion BC of the idealized results of Fig. 3, and he proposed a method of deriving ϕ' and the volumetric strain of the sand at failure v_f from the pressuremeter test data. His expression for the case when $v_f=0$ is identical with the expression derived by

[1] Describing the elastic behaviour in terms of effective stresses.
[2] Note that this is the inverse of the factor N_ϕ.

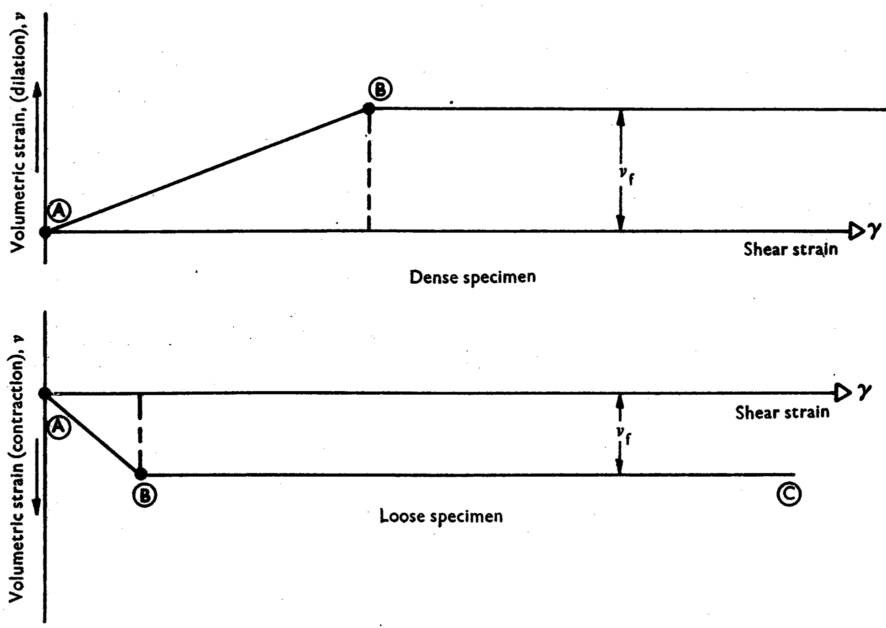

Fig. 3. Idealization of shear behaviour of granular materials as proposed by Ladanyi (1963)

Gibson and Anderson (1961). The volumetric strain of the sand at failure is to be chosen by trial and error so that a straight line is produced when log σ'_{r_a} is plotted against log $[(\Delta V/V) + v_f]$. However, if prior to failure, the sand behaves as an elastic material, the conditions of the pressuremeter test (assumed to be the expansion of an infinitely long cylinder) are such that the volumetric strains are zero. Hence it would be expected in practice that v_f would be nearly zero; indeed, if v_f is taken as zero then the results of the Authors' tests produce a straight line for all but the initial part of the plot of log σ'_{r_a} against log $(\Delta V/V)$. Consequently the method is very sensitive to the values observed for the initial part of the test—and for all practical purposes the results suggest that $v_f = 0$, when the analysis becomes identical with that of Gibson and Anderson (1961).

Vesić (1972) proposed a solution for the general cases of spherical and cylindrical cavities in an infinite soil mass. For the cylindrical case, the method enables the evaluation of pressuremeter tests in terms of a limit pressure, if a value of Poisson's ratio is assumed. The effects of volume changes in the sand are accounted for by using the results of laboratory tests on the same sand at the same density. But, as it is difficult to measure the in situ density of a sand deposit with any accuracy, this solution can have only a limited use in the interpretation of pressuremeter test data.

Schmertmann (1975) reports the work of Al-Awkati in determining ϕ' from pressuremeter test data. Al-Awkati introduces the effects of volume change by using an empirical correlation between the volumetric strain and the relative density. It seems likely that this correlation will not apply generally to sands because of the variety of particle shape and composition. Also, any procedure involving the use of relative density will suffer from significant scatter as suggested by Tavenas and La Rochelle (1972).

From the foregoing it can be seen that in order to derive a value for ϕ' from pressuremeter data some assumptions about the dilatant behaviour of sand must be made. The method proposed in this Paper uses the results of a large number of plane strain shear tests on Leighton Buzzard sand in the simple shear apparatus, interpreted in terms of the theory of stress dilatancy (Rowe, 1962, 1971).

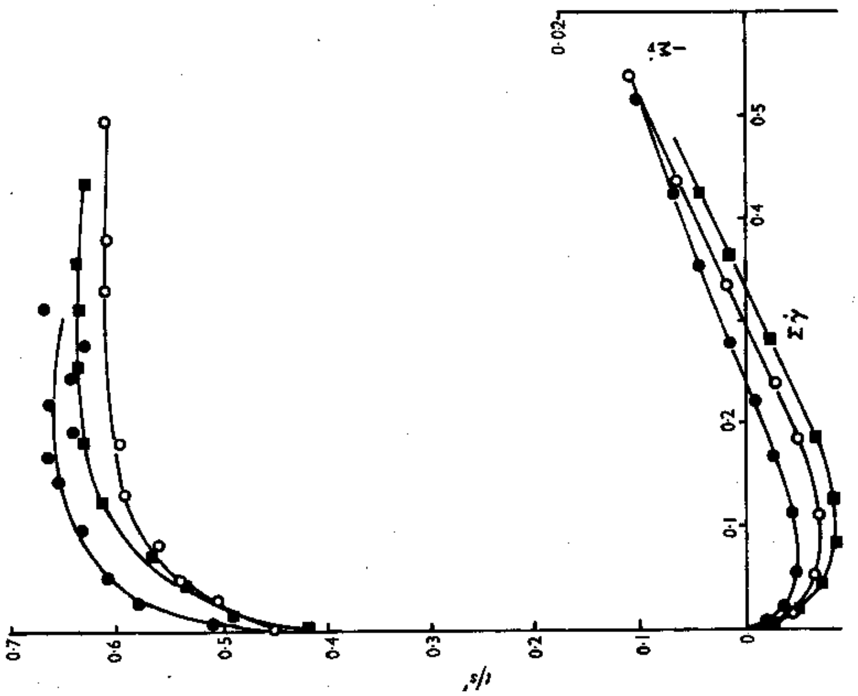

Fig. 5. Results of simple shear tests on loose sand (after Stroud, 1971)

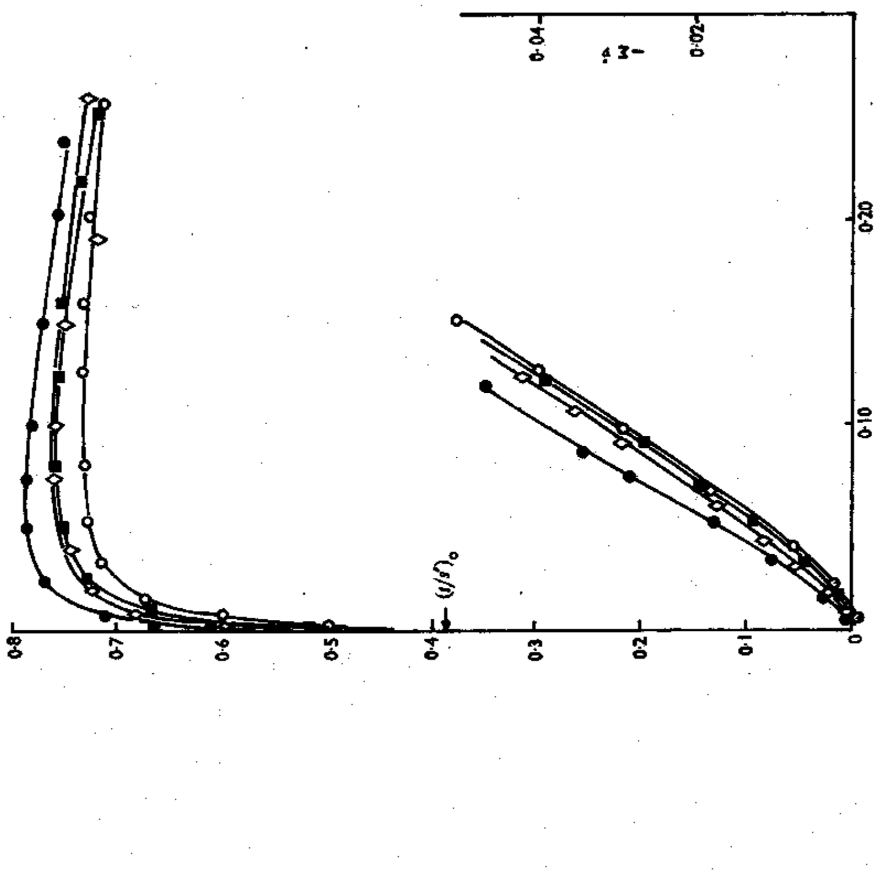

Fig. 4. Results of simple shear tests on dense sand (after Stroud, 1971)

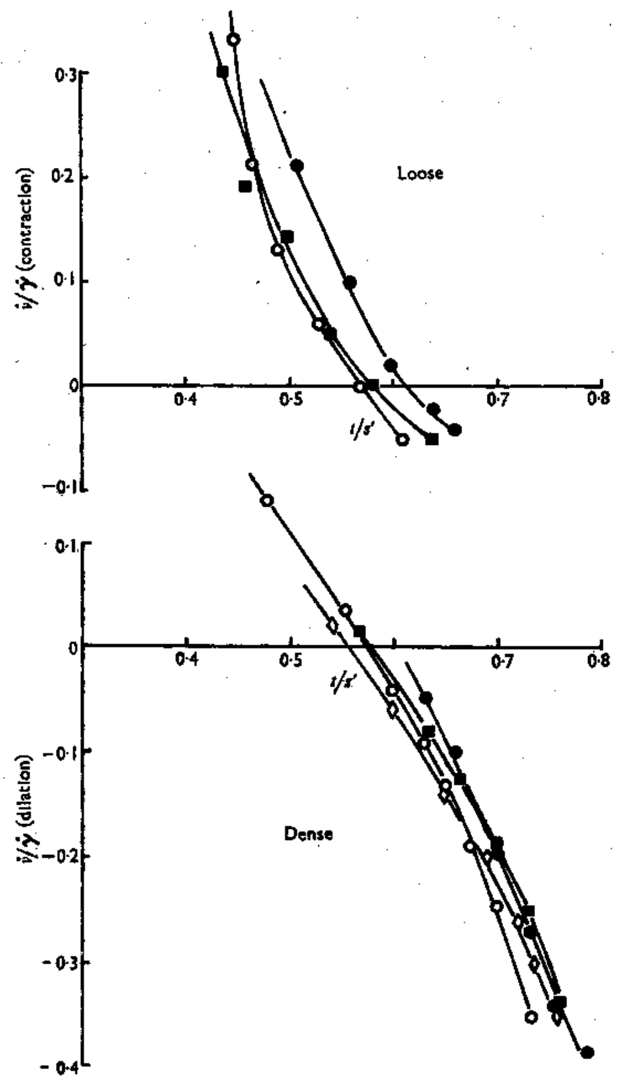

Fig. 6. Dilatation rates from simple shear tests on sand (after Stroud, 1971)

EXPERIMENTAL DATA OF DRAINED PLANE STRAIN TESTS ON SAND

The behaviour of sand under conditions of plane strain can be illustrated with the results obtained by Stroud (1971) in the simple shear apparatus. Fig. 4 shows the results of four different tests on Leighton Buzzard sand in an initially dense condition and Fig. 5 three different tests on initially loose sand. The upper curves are plots of the stress ratio t/s' against cumulative engineering shear strain $\sum\dot{\gamma}$, and the lower curves are the corresponding plots of the volumetric strain $\sum\dot{v}$ against shear strain. The stress parameters used are $s'=\frac{1}{2}(\sigma'_1+\sigma'_3)$ and $t=\frac{1}{2}(\sigma'_1-\sigma'_3)$ where σ'_1 and σ'_3 are the major and minor principal effective stresses. For the strains, compressive values are taken as positive and a dot above a variable (e.g. $\dot{x}$) indicates a small[3] increment.

Each test of the two sets was conducted with a different (but constant) value of the vertical effective stress. For all tests, after the peak stress ratio is reached, the values both of t/s' and $\dot{v}/\dot{\gamma}$ remain sensibly constant over a substantial range of strain, and this experimental observation will be used later in the interpretation of pressuremeter tests. Similar behaviour was

[3] This notation is adopted from the theory of plasticity in which material properties are assumed to be independent of time.

observed in all Stroud's other tests in which the stress levels were varied (as happens in a pressuremeter test). James and Bransby (1970) analysed the strain fields observed throughout a mass of sand behind model retaining walls; these walls were failed in passive modes by rotation separately about the top and the toe. Although the magnitudes of the strains vary significantly throughout the field at any one stage of the test, the ratios of volumetric strain to shear strain do not, as shown by their Fig. 15.

In detail, the variation of the ratio of dilatation $\dot{v}/\dot{\gamma}$ with t/s' for each of Stroud's seven tests is plotted in Fig. 6. These results agree well with Rowe's stress dilatancy theory which relates stress ratio and dilatancy rate in the following manner:

$$R = KD \qquad (2)$$

where $R = \sigma'_1/\sigma'_3$, $K = \tan^2(\pi/4 + \phi'_f/2)$ and $D = 1 - \dot{v}/\dot{\epsilon}_1$. For plane strain conditions, the angle ϕ'_f is taken equal to the angle of internal friction ϕ'_{cv} at the end of a test when all dilatation has ceased, and the sand has reached a critical state, and continues to deform at constant volume.

Equation (2) can be rewritten directly in the form

$$\frac{1+\sin\phi'}{1-\sin\phi'} = \left(\frac{1+\sin\phi'_{cv}}{1-\sin\phi'_{cv}}\right)\left(\frac{1+\sin\nu}{1-\sin\nu}\right) \qquad (3)$$

where $\sin\nu = -\dot{v}/\dot{\gamma}$ and ν is defined as the angle of dilatation (after Hansen, 1958) and is directly analogous to the definition of ϕ'.

It will be assumed (*a*) that eqn (2) uniquely relates values of ϕ' with ν and (*b*) that ϕ' and ν are constant for one particular test after failure has been induced. However, the pair of values (ϕ', ν) will depend on the initial density of the sand, and to a lesser extent on the initial stress level.

ANALYSIS OF DRAINED EXPANSION TEST

The sand surrounding the expanding pressuremeter is assumed to be deformed under conditions of axial symmetry and plane strain. There is good experimental evidence for these two basic assumptions from an experimental study by Laier *et al.* (1975). Further support comes from finite element computations by Hartman and Schmertmann (1975). Consequently, all displacements are in a radial direction only, and it is sufficient to consider a section through the soil perpendicular to the axis of the pressuremeter as in Fig. 7. Cylindrical co-ordinates are used, and the radial displacement of any point is denoted by ξ. Because of axial symmetry, the principal stresses acting on an element of soil are (σ'_r, σ'_θ, σ'_z); initially $\sigma'_r = \sigma'_\theta$ but as the expansion of the cavity (of current radius a) occurs σ'_r increases to become the major stress. The current strains are given by the expressions

$$\epsilon_r = -\frac{d\xi}{dr}, \quad \epsilon_\theta = -\frac{\xi}{r}, \quad \epsilon_z \equiv 0 \qquad (4)$$

where compressive strains have been taken as positive.

During an expansion test at a particular depth all the elements of soil will be subjected to the same stress path but they will have reached different stages on the path at any one instant depending on their radial distance from the pressuremeter. Let the radial effective stress associated with the onset of failure be σ'_R. This pressure will be constant at one depth, that is for any one test, irrespective of the initial diameter of the pressuremeter, but it is likely to increase systematically with depth, and hence for each separate test.

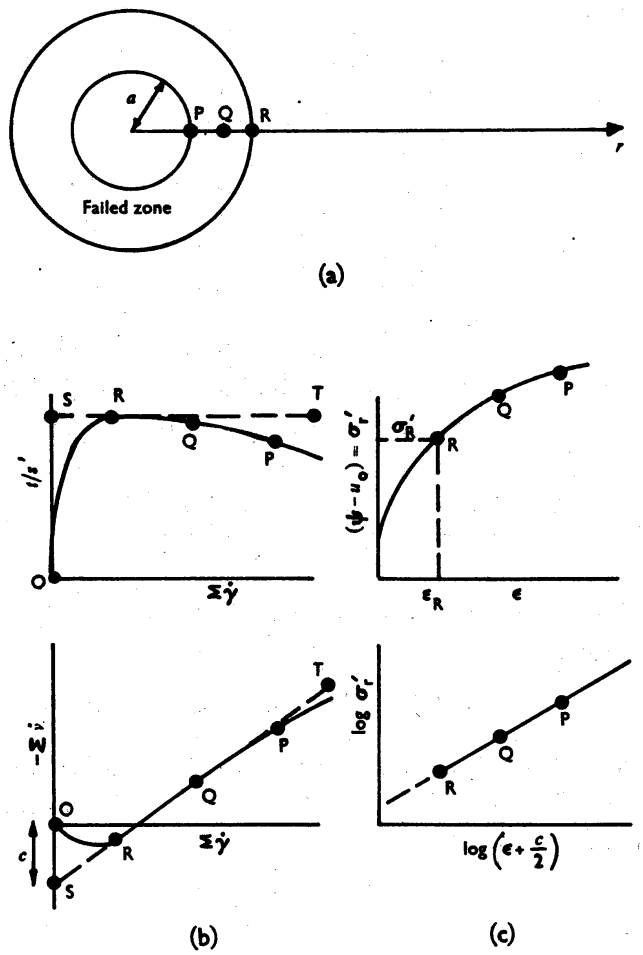

Fig. 7. Stress–strain behaviour of sand around an expanding cylindrical cavity

When the applied pressure ψ is increased above the value $(\sigma'_R + u_0)$, where u_0 is the ambient pore-water pressure, an annular zone of soil, of some outer radius R, which is at the failure stress ratio, will surround the pressuremeter, as sketched in Fig. 7(a). The size of the annulus will grow as the applied pressure is increased further. However, the radial effective pressure σ'_R, and the corresponding strains at the outer boundary $r = R$ of the failed zone will be unchanged regardless of the size R of the zone.

Consider the conditions inside the failed zone $(a < r < R)$ at one stage of the pressuremeter test. The stress–strain relationships for the sand are given by the full line curves ORQP in Fig. 7(b); the dashed straight lines SRT represent the idealized behaviour. At point R, the sand has just reached failure, whereas points Q and P are at states further along the stress–strain curve. All the sand is assumed to be at the failure stress ratio, given by line SRT, so that

$$\frac{\sigma'_\theta}{\sigma'_r} = N = \frac{1-\sin\phi'}{1+\sin\phi'} = \text{constant} \quad (0 < N < 1) \qquad (5)$$

The differential equation of equilibrium[4] which must be satisfied is

$$\frac{d\sigma'_r}{dr} + \frac{\sigma'_r - \sigma'_\theta}{r} = 0 \qquad (6)$$

[4] Since there are no excess pore-pressures, the equation is satisfied by effective stresses, as well as total stresses.

Substituting for σ'_θ from eqn (5), and rearranging

$$-\frac{d\sigma'_r}{\sigma'_r} = (1-N)\frac{dr}{r} \quad \ldots \quad (7)$$

Integrating eqn (7) and using the outer boundary condition that $\sigma'_r = \sigma'_R$ at $r = R$

$$\ln\frac{\sigma'_r}{\sigma'_R} = (1-N)\ln\frac{R}{r} \quad \ldots \quad (8)$$

This equation governs the radial distribution of the radial effective stress within the annular zone of failed sand. The sand is also assumed to be failing such that the relationship between volumetric and shear strains is given by the straight line SRT. Along this line, the dilatation rate is constant so that

$$\sin\nu = -\frac{\dot{v}}{\dot{\gamma}} = -\frac{(\dot{\epsilon}_r+\dot{\epsilon}_\theta)}{(\dot{\epsilon}_r-\dot{\epsilon}_\theta)} \quad \ldots \quad (9)$$

In general, this line will not necessarily pass through the origin so that the cumulative strains will be related by the expression

$$-\Sigma\dot{v} = (\Sigma\dot{\gamma})\sin\nu - c \quad \ldots \quad (10)$$

where S is considered to be below O, representing a positive compressive volumetric strain, c.

Since there is no rotation of principal strains $\Sigma\dot{v} = v = (\epsilon_r+\epsilon_\theta)$ and $\Sigma\dot{\gamma} = \gamma = (\epsilon_r - \epsilon_\theta)$; eqn (10) can be rewritten in the form

$$-\epsilon_r = n\epsilon_\theta - \frac{c(n+1)}{2} \quad \ldots \quad (11)$$

in which

$$n = \frac{1-\sin\nu}{1+\sin\nu} = \text{constant} \quad (0<n<1) \quad \ldots \quad (12)$$

and is directly analogous to the definition of N.

Substituting for ϵ_r and ϵ_θ in eqn (11) from the basic definitions of strain (eqn (4)) leads to the differential equation

$$\frac{d\xi}{dr} = -n\frac{\xi}{r} - \frac{c(n+1)}{2} \quad \ldots \quad (13)$$

which governs the radial distribution of the displacement within the annular zone of failed sand.

Multiplying through eqn (13) by r^n and integrating gives

$$\xi r^n = -\frac{c}{2}r^{n+1} + \text{constant} \quad \ldots \quad (14)$$

Using the outer boundary condition that at $r = R$, $\xi/r = \xi_R/R = \epsilon_R = \text{constant}$, then

$$\xi r^n = -\frac{c}{2}r^{n+1} + \epsilon_R R^{n+1} + \frac{c}{2}R^{n+1}$$

or

$$\frac{\xi}{r} = \left(\frac{R}{r}\right)^{n+1}\left(\epsilon_R + \frac{c}{2}\right) - \frac{c}{2} \quad \ldots \quad (15)$$

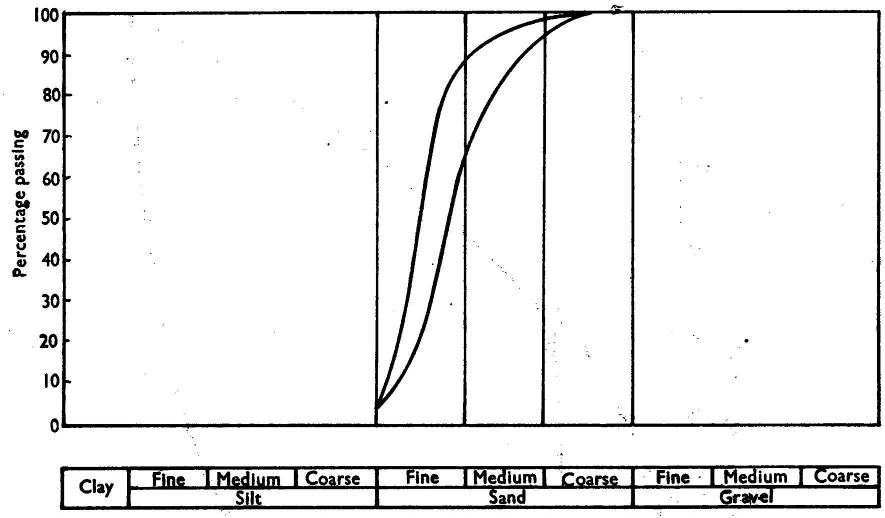

Fig. 8. Particle size distribution of the Wash sand

Eliminating (R/r) from eqns (8) and (15) gives

$$\left(\frac{\xi}{r}+\frac{c}{2}\right) = \left(\epsilon_R+\frac{c}{2}\right)\left(\frac{R}{r}\right)^{n+1} = \left(\epsilon_R+\frac{c}{2}\right)\left(\frac{\sigma'_r}{\sigma'_R}\right)^{(n+1)/(1-N)} \quad . \quad . \quad . \quad . \quad (16)$$

Eqn (16) is valid throughout the annular zone; in particular at the inner boundary $r=a$ where the measurements are made

$$\left(\epsilon+\frac{c}{2}\right) = \left(\epsilon_R+\frac{c}{2}\right)\left(\frac{\psi-u_0}{\sigma'_R}\right)^{(n+1)/(1-N)} \quad . \quad . \quad . \quad . \quad . \quad . \quad (17)$$

where $\epsilon=\xi_a/a$ is the strain measured during a pressuremeter test.

Taking logarithms,

$$\log\left(\epsilon+\frac{c}{2}\right) = \frac{n+1}{1-N}\log(\psi-u_0)+\text{constant} \quad . \quad . \quad . \quad . \quad . \quad (18)$$

so that a linear relationship is predicted if $\log(\psi-u_0)$ is plotted against $\log[\epsilon+(c/2)]$ in Fig. 7(c). The gradient of this straight line, denoted s, is given by

$$\frac{1-N}{n+1} = \frac{(1+\sin\nu)\sin\phi'}{1+\sin\phi'} = s \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (19)$$

This equation may be solved with eqn (3)

$$\frac{1+\sin\phi'}{1-\sin\phi'} = K\frac{1+\sin\nu}{1-\sin\nu} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3\text{ bis})$$

to produce

$$\sin\phi' = \frac{(K+1)s}{(K-1)s+2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (20)$$

$$\sin\nu = \frac{2Ks-(K-1)}{K+1} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (21)$$

The problem arises as to what value to assign to the constant c, which is not measured experimentally. Fortunately, the value turns out to be sufficiently small to be negligible, both

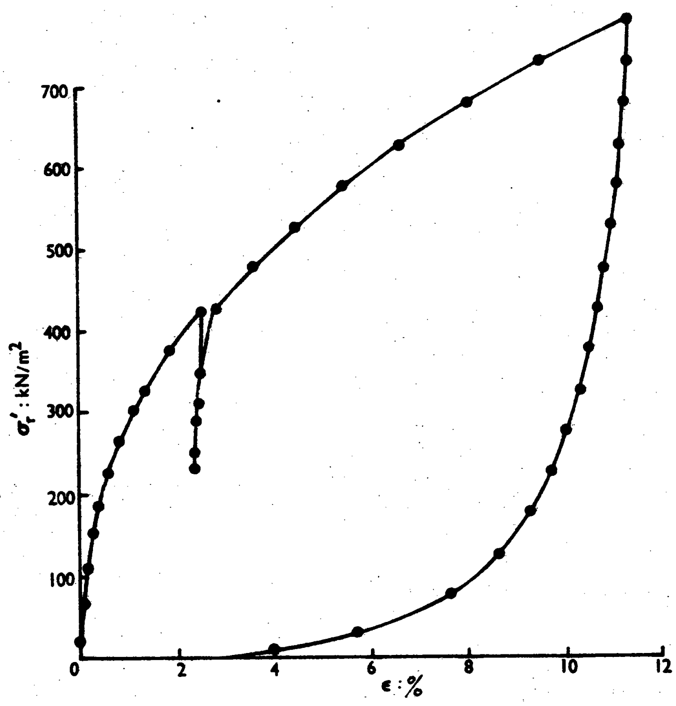

Fig. 9. Results of a pressuremeter test in the Wash sand

as a result of a theoretical analysis set out in the Appendix, and from a study of the data from pressuremeter tests presented in the next section.

TEST RESULTS FROM THE WASH SANDS

A series of tests has been conducted in the Wash sands in a number of boreholes, close to the Tidal Defence Embankment, in an area opposite the Inshore Trial Embankment built as part of the Wash Water Storage Feasibility Study. The sand has a typical particle size distribution as shown in Fig. 8. Nearby borings at the Inshore Trial Bank site (Wimpey Laboratories Ltd, 1973) indicate that the sand is loose, becoming medium dense to dense, containing occasional pockets and partings of organic silt.

The results of an expansion test at a depth of 7 m are shown in Fig. 9. After the pressuremeter was inserted to the desired depth, the membrane was expanded by applying discrete pressure increments and waiting until the radial expansion had come to equilibrium, before applying the next increment. The applied pressure has been corrected by a (constant) amount of 14 kN/m^2 to allow for the strength of the rubber membrane and protective metal sheath. Since no excess pore-pressures develop in the sand, the conditions of the test are fully drained and the corrected pressure ψ equals the sum of the effective radial pressure σ'_r (in the sand in contact with the membrane) and the ambient pore-pressure u_0.

The form of the curve is similar to that obtained from undrained tests in clays, except that the results for sand do not approach a limit pressure. At the end of the test the membrane collapses back to its initial size at zero radial effective stress. This has been observed in all tests conducted in sands, and is considered to be a good way of estimating the position of the water table. The test results have also been plotted as σ'_r against ϵ on logarithmic scales in Fig. 10. Clearly, after a strain of 1·4%, the results lie on a well-defined straight line. Other test results from this site lie on similarly well-defined lines. If the value of c, defined in Fig. 7, is taken as negligible then s is equal to the gradient of the lines. To deduce values of ϕ' and ν from an observed value of s, a value of ϕ'_{cv} or K must be measured or estimated.

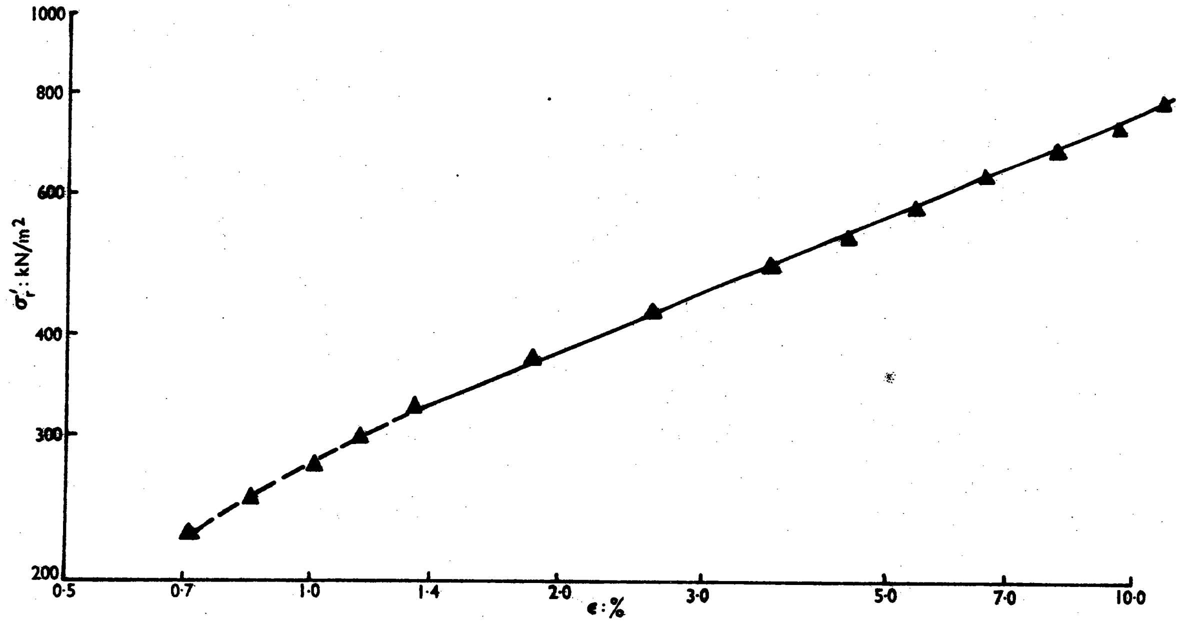

Fig. 10. Results of a pressuremeter test in the Wash sand

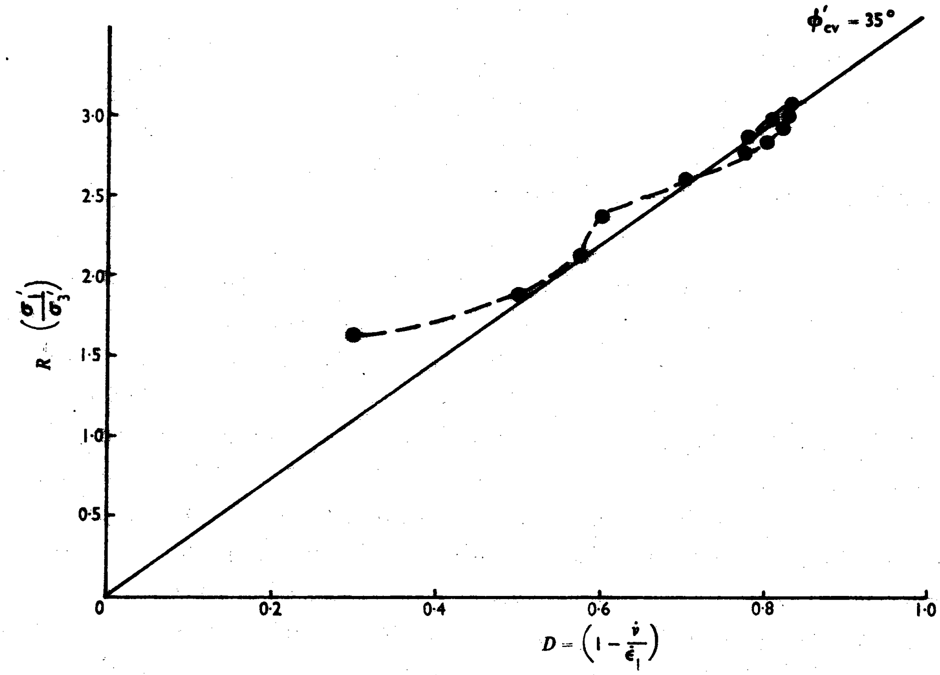

Fig. 11. Stress–dilatancy plot of drained triaxial compression test on Wash sand

Drained triaxial compression tests have been performed on a number of samples of the sand (Soil Mechanics Ltd, 1971). The samples were prepared in a loose state by pouring under de-aired water, and tested under a back pressure. It is possible to deduce values of ϕ'_{cv} from these tests on loose samples by plotting stress ratio R against dilatancy D. One such plot is shown in Fig. 11; the close agreement between the results and the simple version of the theory for the initial stages of the test is atypical. The other test results all produce similar values for ϕ'_{cv}, of which the average is 35°, giving a value for K of 3·68.

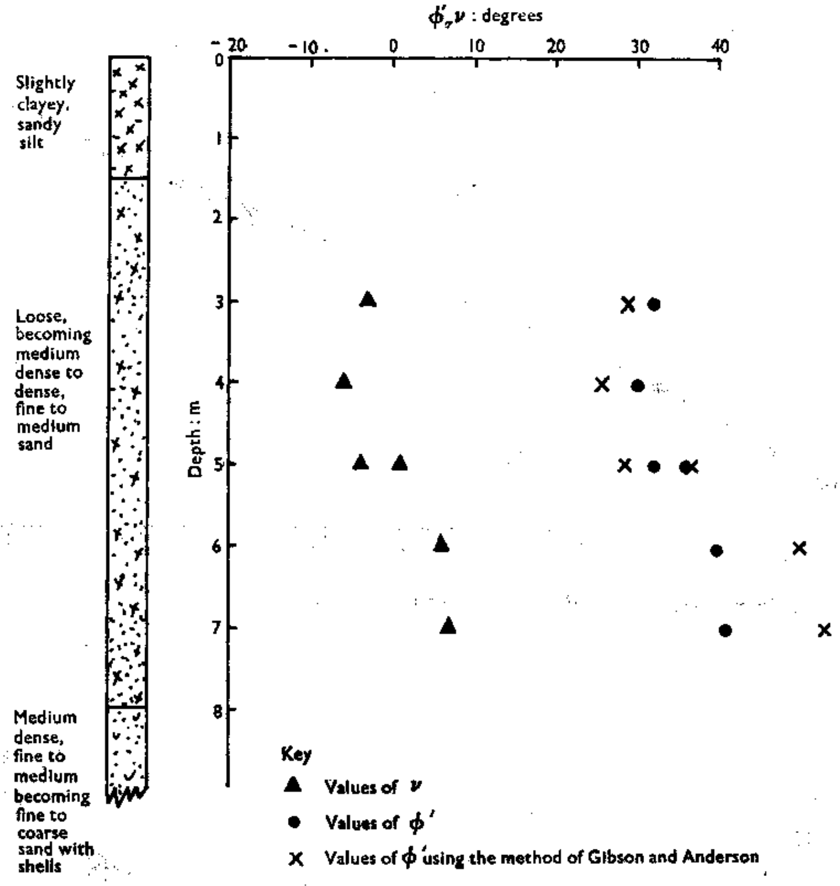

Fig. 12. Derived values of ϕ' and ν for the Wash sand

The laboratory samples may well have been tested at inappropriate voids ratios, as the in situ density is not known with any accuracy. This would mean that the laboratory value of ϕ' of 33° may not be relevant to field conditions. However, the angle ϕ'_{cv} is a property of the sand grains, supposedly independent of the voids ratio. It is necessary only to test loose samples of the sand at the appropriate stress level to derive a value for ϕ'_{cv}. Rowe (1971) noted that measured values of ϕ'_{cv} depend on the stress level and on the surface treatment of the soil particles. As disturbed samples of the natural sand were taken and reconsolidated to the in situ stress state these two factors will have been taken into account.

Equations (20) and (21) have been solved for the series of tests with ϕ'_{cv} taken as 35°. The corresponding values of ϕ' and ν are shown for the various tests against depth alongside a borehole log in Fig. 12. Eqns (3) and (19) can also be solved graphically. This is instructive in that it shows the solution to be mathematically well-conditioned. This has been done for two test results and is shown in Fig. 13.

The pressuremeter values of ϕ' are derived from a plane strain test and are not directly comparable with the value of ϕ' of 33° derived from triaxial tests. However, for loose sand, these two sets of values should be close. It can be seen that the agreement is good for the specimens taken from near the surface becoming less good with depth. This pattern is due to the increasing density of the sand with depth.

The results have also been analysed, using the method of Gibson and Anderson (1961). The resulting values of ϕ', included in Fig. 12, are very much as expected. Their method assumes that no volume change occurs during the deformation, which is unrealistic. It under-estimates

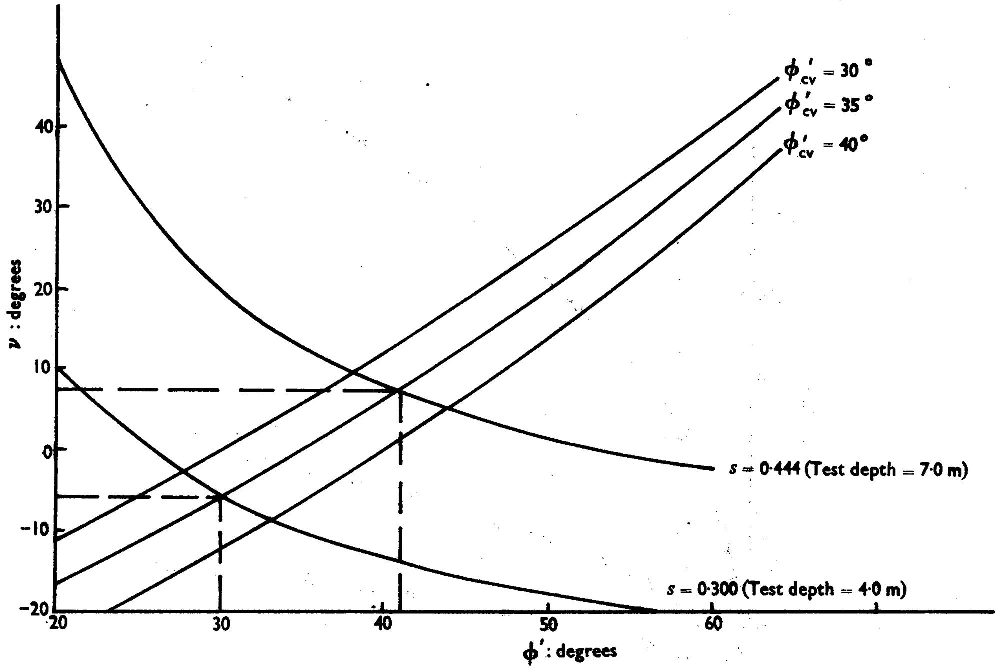

Fig. 13. Graphical method of solution for values of φ′ and ν

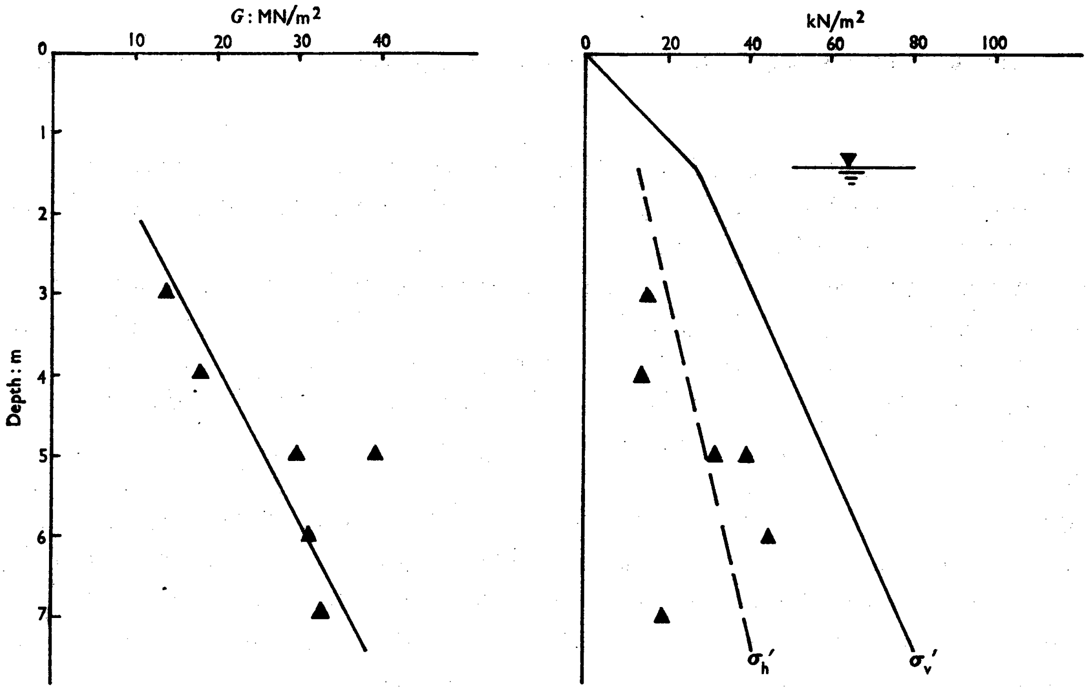

Fig. 14. Values of shear modulus and in situ lateral effective stress for the Wash sand

the value of ϕ' when the sand is compacting and over-estimates the value when the sand is dilating. As mentioned previously, it was not possible to assign values for v_f other than zero, and thus Ladanyi's (1963) method was not used to evaluate the test data because the solutions would have become identical with those of Gibson and Anderson (1961).

Values of shear modulus G have been obtained from the reloading cycles in the pressuremeter tests. It is not possible to determine values for Young's modulus as the Poisson's ratio is not known. The moduli have been derived over a stress increment of 100 kN/m². This size of stress increment appears to give reasonable values for London Clay (Windle and Wroth,

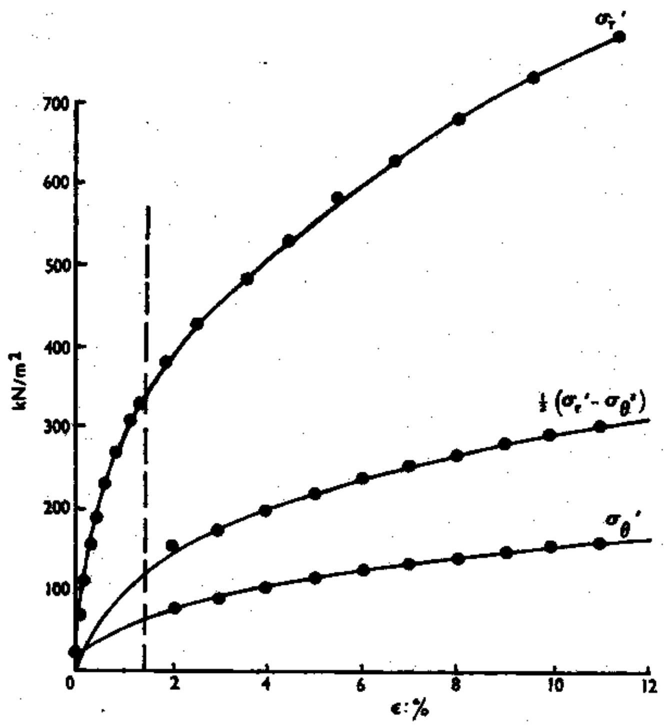

Fig. 15. Derived values of the principal effective stresses during a pressuremeter test in the Wash sand

1977). Initial secant moduli have been evaluated over a number of stress and also strain increments, but they are not reported because they are too scattered to be of value. The reload moduli are shown in Fig. 14.

Values of lateral effective stress σ'_h have been determined by observing the pressure at which the membrane starts to move. They are shown in Fig. 14 where they are compared with a plot of vertical effective stress σ'_v. The value of σ'_v was estimated using a value of 19 kN/m³ for the bulk weight of the sand, derived from results obtained by use of a Bishop sand sampler (Soil Mechanics Ltd, 1971). The values of σ'_h are very scattered, but indicate that K_0 is about 0·5.

The observed values of initial modulus and σ'_h are very sensitive to the details of the drilling technique and type and position of the cutter. These should be altered as the density of the sand deposit changes and it is not possible, at present, to do this in a proper manner. Consequently, values of initial modulus and lateral effective stress observed to date are subject to some scatter.

It is possible to plot the complete stress–strain data for the sand after failure has been induced. The values of σ'_r and ϵ are measured during the test, and σ'_θ can be derived. Thus σ'_θ and $\frac{1}{2}(\sigma'_r - \sigma'_\theta)$ can be plotted against ϵ. This has been done for the typical test result of Fig. 9 and is shown in Fig. 15. The curves of σ'_θ and $\frac{1}{2}(\sigma'_r - \sigma'_\theta)$ have been assumed to be smooth for $0 < \epsilon < 1{\cdot}4\%$ and have been simply sketched in.

Unfortunately the method of analysis does not produce sensible values for pressuremeter test results from disturbed sands. While testing different types and positions of the cutter a number of pressuremeter tests was conducted on disturbed sand. A typical test result is shown in Fig. 16. The observed value for lateral effective stress of 14 kN/m² is low. The value of the shear modulus of 15 MN/m² observed during the reloading cycle appears reasonable, although this might be a coincidence. The values for ϕ' and ν of 52° and 24° however,

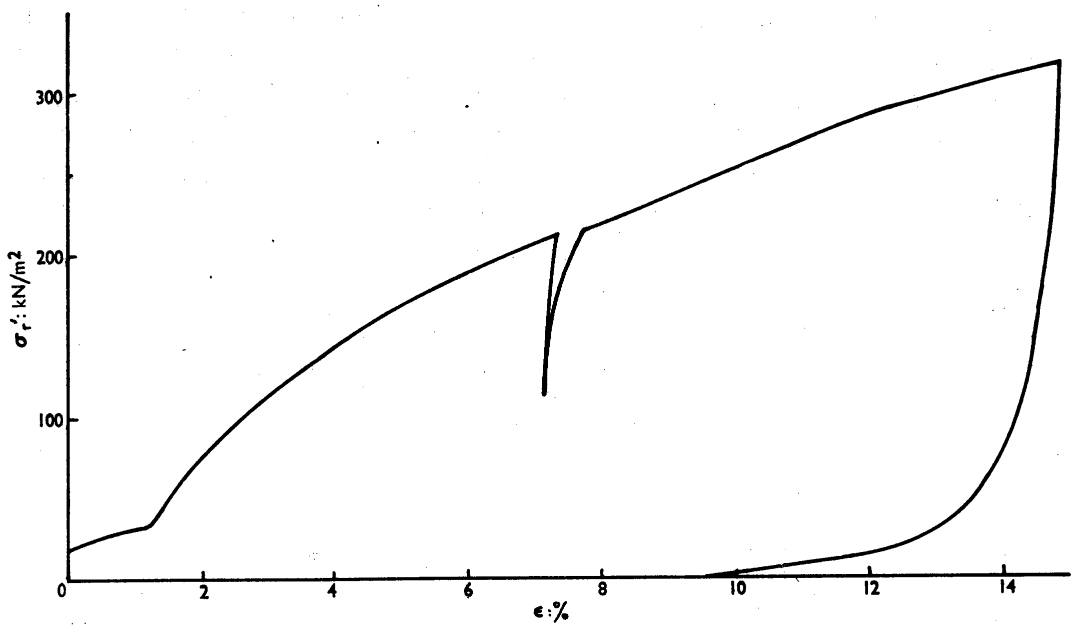

Fig. 16. Results of a pressuremeter test in disturbed Wash sand

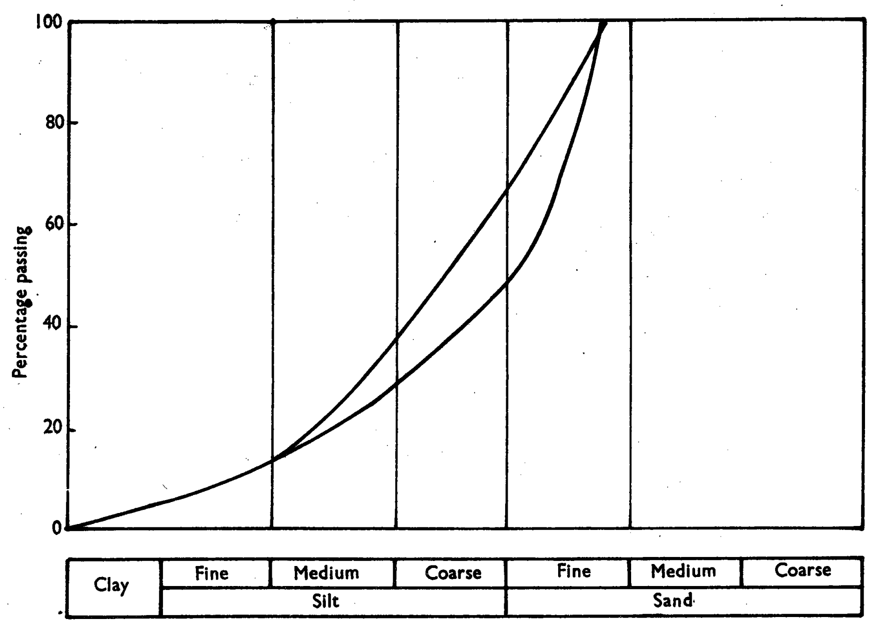

Fig. 17. Particle size distribution of the hydraulic fill from Kernick

are very high, unrealistically so. Similarly high values of ϕ' and ν have been observed in other tests, where the cutter had subsequently been discovered to have been too far forward. The analysis is inapplicable because the sand is not following a unique stress–strain curve and weak inner annuli of sand are interacting with stronger outer annuli of sand.

TEST RESULTS FROM KERNICK

The Kernick site has been described by Appleton (1974) and by Illsley *et al.* (1976). It consists of a large earth and rock-fill dam, behind which a fine micaceous sand waste has been deposited as hydraulic fill. Consequently, the sand is in a loose condition and is well graded. In the area tested, the material has a typical grading curve, shown in Fig. 17. At the time of the investigation, many problems related to inserting the self-boring pressuremeter in stiff materials had not been overcome. Consequently this soft, somewhat unusual, sandy-silt material received more attention than would normally be the case because it permitted more accurate pressuremeter tests to be made. Typical pressuremeter test curves are not shown as they are similar to the curve shown in Fig. 9. The results have all been plotted in the form log σ'_r against log ϵ, and values for the gradient s determined. A number of drained triaxial tests has been performed on dense and loose disturbed samples: a selection of the results is

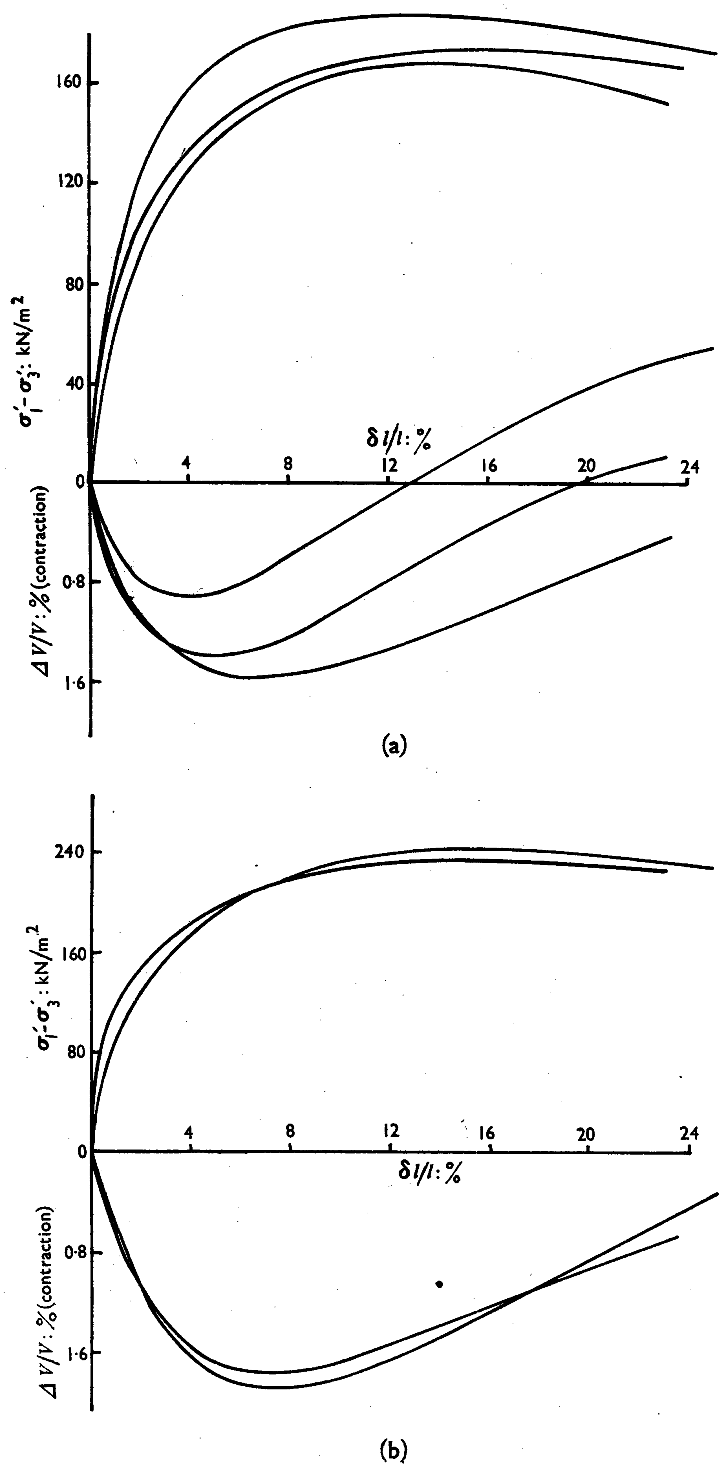

Fig. 18. Results of drained triaxial compression tests on loose samples of Wash sand at cell pressures of: (a) 60 kN/m^2; (b) 80 kN/m^2

shown in Fig. 18. The test results were also plotted as stress ratio against dilatancy. Early in the tests there is poor agreement with Rowe's theory, possibly because the micaceous particles bend rather than slip past one another as they are deformed. However, at larger strains, there is good agreement and the tests all produce the same value for ϕ'_{cv}, namely 39°. This is reasonable because the sand is exceptionally angular, having been produced during the kaolinization of granite and having remained essentially unweathered. This value for ϕ'_{cv} was used to determine the values of ϕ' and ν plotted in Fig. 19. The values are believed to be appropriate for a hydraulic fill of angular sand.

An initial estimate, using $\phi'_{cv}=35°$ resulted in preliminary values of ϕ' consistently 3° too large and values of ν consistently 3° too small. However, this very angular sand represents an extreme case, and it is suggested that, for quartz sands, a satisfactory estimate for engineering purposes can be obtained using a value of $\phi'_{cv}=35°$ if no direct measurements are available.

Dutch cone soundings, using a Delft Mantle cone, were performed close to the borehole (Fig. 20). These may be used to determine approximate values of ϕ' for the sand according to a variety of methods. The resulting values for ϕ' are compared with those obtained from the pressuremeter tests and from the triaxial tests on loose samples of sand in Fig. 21. The agreement between the pressuremeter results and the triaxial tests is very close and better than between any of the methods of interpreting the Dutch cone test data. The good agreement can be attributed partly to the fact that plane strain and triaxial tests give similar values of ϕ' for loose sand. Also the laboratory samples will have been tested at an appropriate voids

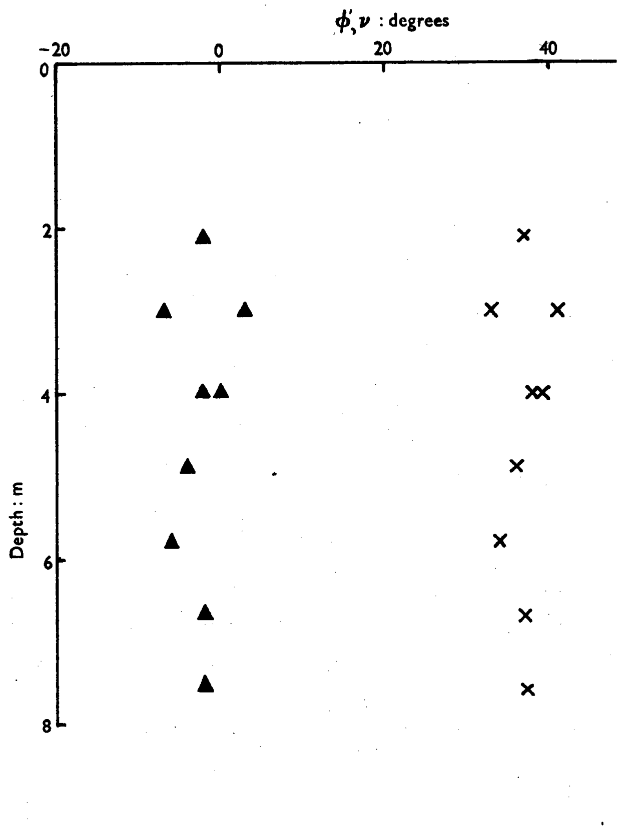

Fig. 19. Derived values of ϕ' and ν for the hydraulic fill at Kernick

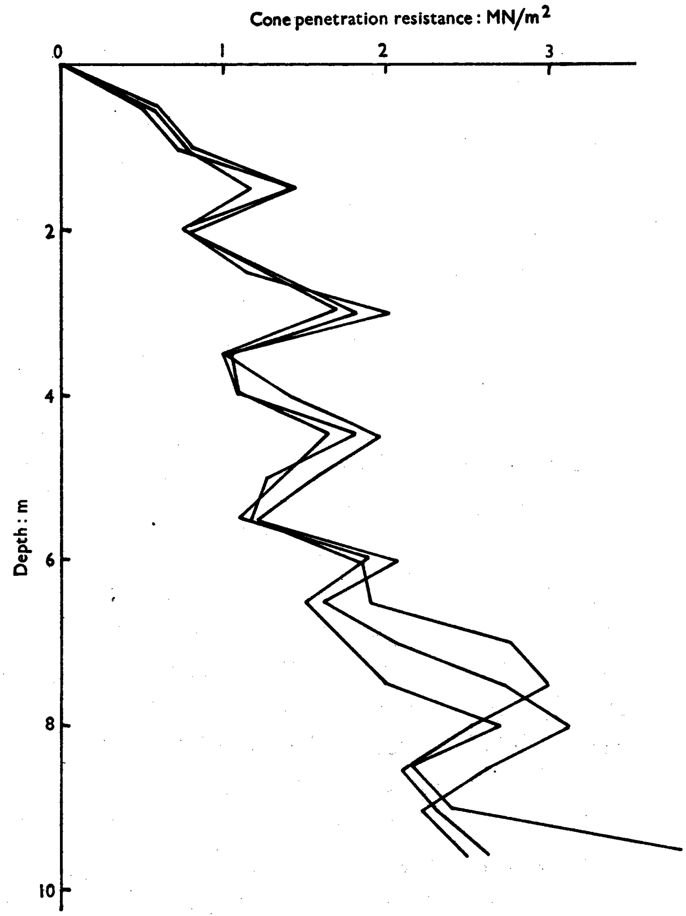

Fig. 20. Results of static penetration tests in the hydraulic fill at Kernick

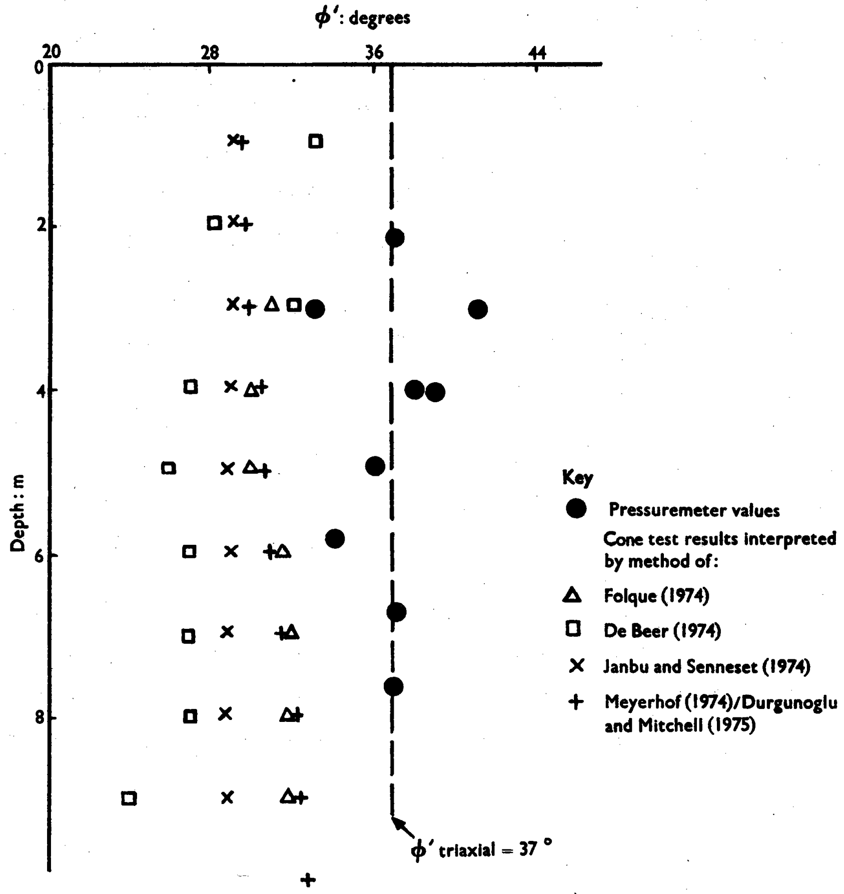

Fig. 21. Comparison of derived values of ϕ' from: (a) static penetration tests; (b) pressuremeter tests; (c) triaxial tests on the hydraulic fill at Kernick

ratio as both samples and the in situ material were deposited under water and have not been overconsolidated. Thus the values of ϕ' from the triaxial tests will be more relevant than is normally the case.

Values of shear modulus G have been obtained as for the Wash sand, and are plotted in Fig. 22; this time a stress increment of 50 kN/m^2 was used instead of 100 kN/m^2, in order that a similar size of strain increment was considered. Values of lateral effective stress are also shown in Fig. 22, where they are compared with a plot of vertical effective stress. The dry unit weight of the sand was estimated to be 11·5 kN/m^3 from the initial voids ratios of the triaxial specimens. This value agrees with a value determined in situ by Baker and Sharrocks (1974). The appropriate moisture content w is unknown and a value of 10% has been selected. These values have been used to estimate the vertical effective stresses. The results indicate that, below about three metres, the material has a K_0 of 0·4. This value agrees closely with that given by the formula $K_0 = 1 - \sin \phi'$ for normally consolidated clays or loose sands, derived from Jaky (1944).

CONCLUSIONS

A new method of determining the angle of internal friction ϕ' and the angle of dilatation ν from pressuremeter test data has been presented. This method incorporates the effects of the volume changes that occur in the sand as it is sheared by using experimental observations of ϕ'_{cv} and the theory of stress-dilatancy.

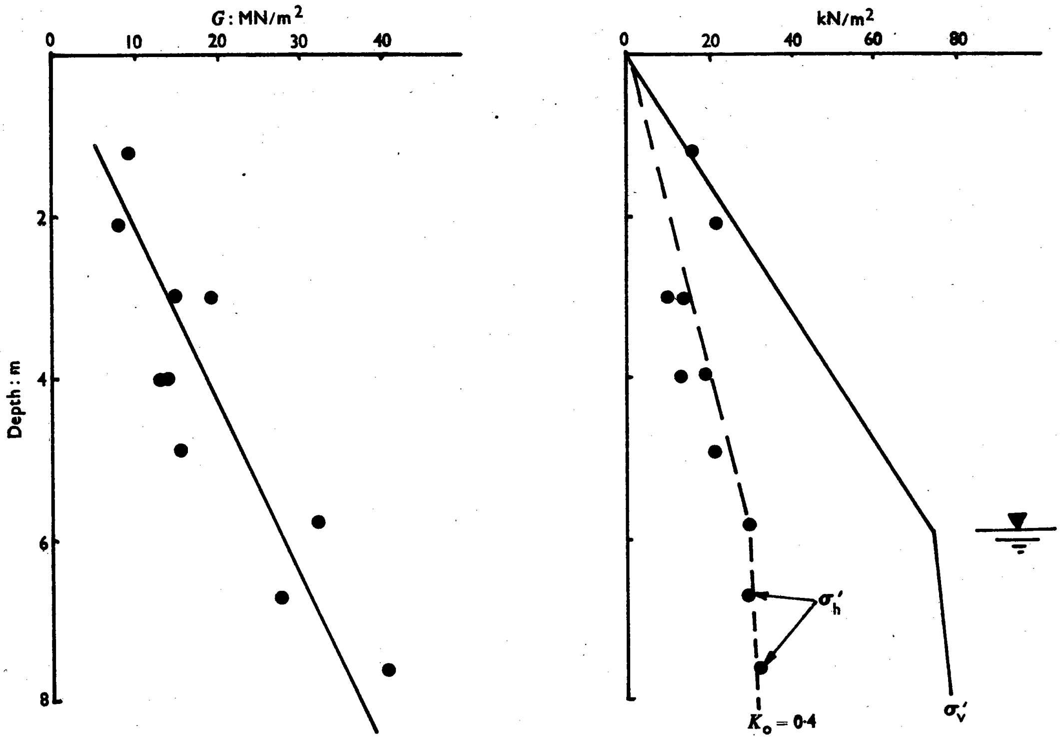

Fig. 22. Values of shear modulus and in situ lateral effective stress for the hydraulic fill at Kernick

The values of ϕ' and ν obtained from typical test results using the new method of analysis agree well with the results of triaxial tests if allowance is made for the fact that the pressuremeter values of ϕ' are plane strain values. As expected the method of analysis does not give sensible values of ϕ' and ν from data of tests conducted on disturbed sand deposits. However, this method does appear to be an improvement over other in situ methods of determining ϕ' because it is able to assimilate correctly the effects of stress level, pore-pressure, stress history, etc.

Values of lateral effective pressure and shear modulus are also presented for the two test sites. The values of shear modulus appear reasonable, but the values of lateral effective pressure indicate that this parameter is very sensitive to any slight disturbance caused during insertion of the pressuremeter.

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The field tests, described in this Paper, have been conducted at two sites, and have used ancillary equipment at one of these sites. The Authors are particularly grateful for the help of the following people and organizations: Mr T. R. M. Wakeling of Foundation Engineering Ltd for help in the provision of some of the field equipment; Binnie and Partners and their clients, the National Water Council, for permission to quote data from site investigation reports concerning the Wash Water Storage Feasibility Study; the Directors of ECLP and Co.

Ltd for permission to perform tests in the hydraulic fill at the site of Kernick dam, and to the company staff and their consultants, Mr T. R. M. Wakeling of Foundation Engineering Ltd and Mander Raikes and Marshall for practical assistance and helpful discussions.

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APPENDIX

In the new analysis of a drained expansion test presented in the Paper, it is assumed that sand fails in conditions of plane strain at a constant stress ratio and at a constant rate of dilatation. The behaviour of the sand prior to failure only affects the interpretation by means of eqn (18) through the value of the constant c, which is defined by eqn (10) and in Fig. 7.

The value of c is now examined assuming that the sand behaves elastically prior to failure. At the start of the pressuremeter test the radial and circumferential stresses are equal and

$$\sigma'_r = \sigma'_\theta = \sigma'_{r0} = \sigma'_{\theta 0} = K_0 \sigma'_{v0} \qquad (22)$$

where σ'_{v0} is the in situ vertical effective stress.
Since the deformation of the sand is assumed to be in conditions of axial symmetry and plane strain, then it follows from Hooke's laws that

$$\left.\begin{aligned} \Delta\sigma'_r &= -\Delta\sigma'_\theta, \quad \Delta\sigma'_z = 0, \\ \Delta\epsilon_r &= -\Delta\epsilon_\theta, \quad \Delta V/V = 0 \end{aligned}\right\} \qquad (23)$$

In particular $\Delta\sigma'_r = 2G\epsilon_r$.

At point R in Fig. 7 the sand has just reached failure when the stress ratio is such that:

$$\frac{1-\sin\phi'}{1+\sin\phi'} = N = \frac{\sigma'_\theta}{\sigma'_r} = \frac{\sigma'_{\theta 0}+\Delta\sigma'_\theta}{\sigma'_{r0}+\Delta\sigma'_r} \qquad (24)$$

But since $\sigma'_{r0} = \sigma'_{\theta 0}$ and $\Delta\sigma'_r = -\Delta\sigma'_\theta$ it follows that

$$\sigma'_r = \sigma'_{r0}(1+\sin\phi') = \sigma'_R \qquad (25)$$

and

$$\epsilon = -\epsilon_\theta = \epsilon_r = \frac{\Delta\sigma'_r}{2G} = \frac{\sigma'_{r0}\sin\phi'}{2G} = \epsilon_R \qquad (26)$$

The equation of the line PQRS idealizing the volumetric strain–shear strain relationship is given by

$$-\Sigma\dot{v} = (\Sigma\dot{\gamma})\sin\nu - c \qquad (27)$$

But at point R, the volumetric strain is zero, as a consequence of the elastic behaviour, so that

$$c = (\Sigma\dot{\gamma})_R \sin\nu = 2\epsilon_R \sin\nu \qquad (28)$$

Hence the adjustment to the strain datum for the logarithmic plot of Fig. 7(c) resulting from eqn (17) is

$$\frac{c}{2} = \epsilon_R \sin\nu = \frac{\sigma'_{r0}\sin\phi'\sin\nu}{2G} \qquad (29)$$

For the Wash sand, typical values for a depth of 5·75 m are $G = 30$ MN/m², $\sigma'_{r0} = 30$ kN/m², $\nu = 5°$, $\phi' = 40°$ giving

$$\frac{c}{2} \simeq \frac{30\times 0{\cdot}643\times 0{\cdot}087}{2\times 30\times 10^3} = 0{\cdot}0028\% \qquad (30)$$

This value is negligible, and it is believed to be representative of the situation in sands generally. The assumption that the adjustment to the strain datum is negligible is borne out both by the experimental data and the above theoretical analysis.

In passing, it should be noted that using the above figures the strain required to induce failure in the sand in contact with the membrane of the pressuremeter would be

$$\epsilon_R = \frac{\sigma_{r0}'\sin\phi'}{2G} \simeq 0{\cdot}03\% \qquad (31)$$

This value is extremely small in terms of the pressuremeter test, and emphasizes the importance of minimizing disturbance for obtaining accurate values of soil parameters and particularly measurements of K_0.

REFERENCES

Appleton, B. (1974). *New Civil Engineer*, 25th July, pp. 19–20.

Arthur, J. R. F., James, R. G. & Roscoe, K. H. (1964). The determination of stress fields during plane strain of a sand mass. *Géotechnique* **14**, No. 4, 283–308.

Baguelin, F. & Jézéquel, J. F. (1975). Further insights on the self-boring technique developed in France. Session IV, *Proc. Am. Soc. Civ. Engrs Specialty Conf. In Situ Measurement of Soil Properties*, Raleigh, North Carolina **II**, 231–243.

Baguelin, F., Jézéquel, J. F. & Le Méhauté, A. (1974). Self-boring placement method of soil characteristics measurement. *Proc. Specialty Conf. Subsurface Exploration for Underground Excavation and Heavy Construction*, Henniker, pp. 312–332.

Baker, M. T. & Sharrocks, D. M. (1974). *Report of a site investigation carried out at Kernick dam.* Research Report, Cambridge University Engineering Department.

Bratchell, G. E., Leggatt, A. J. & Simons, N. E. (1974). The performance of two large oil tanks founded on compacted gravel. *Proc. Conf. Settlement of Structures, Cambridge.* London: Pentech Press.

De Beer, E. (1974). Interpretation of results of static penetration tests. *Proc. European Symp. Penetration Testing, Stockholm* **2.1**, 62–72.

Durgunoglu, H. T. & Mitchell, J. K. (1975). Static penetration resistance of soils. *Proc. Am. Soc. Civ. Engrs Specialty Conf. In Situ Measurement of Soil Properties*, Raleigh, North Carolina **I**, 151–189.

Folque, J. (1974). Penetration testing in Portugal. *Proc. European Symp. Penetration Testing, Stockholm* **1**.

Gibbs, H. J. & Holtz, W. G. (1957). Research on determining the density of sands by spoon penetration testing. *Proc. 4th Int. Conf. Soil Mech. London* **1**, 35–39.

Gibson, R. E. & Anderson, W. F. (1961). In situ measurement of soil properties with the pressuremeter. *Civ. Engng Pub. Wks Rev.* **56**, No. 658, May, 615–618.

Hansen, B. (1958). Line ruptures regarded as narrow rupture zones. Basic equations based on kinematic considerations. *Proc. Conf. Earth Pressure Problems, Brussels* **1**, 39–48.

Hartman, J. P. & Schmertmann, J. H. (1975). FEM study of elastic phase of pressuremeter test. *Proc. Am. Soc. Civ. Engrs Specialty Conf. In Situ Measurement of Soil Properties*, Raleigh, North Carolina **I**, 190–207.

Hughes, J. M. O. (1973). *An instrument for in situ measurement in soft clays.* PhD thesis, University of Cambridge.

Illsley, A. E., Wakeling, T. R. M. & Humphreys, J. D. (1976). The Kernick and Portworthy mica residue disposal schemes for E.C.L.P. in Cornwall and South Devon, U.K. *12th Int. Conf. on Large Dams*, Q.44, R.22, pp. 437–461.

Jaky, J. (1944). The coefficient of earth pressure at rest. *Magy. Mérn.-és Épit.-Egyl. Közl.*

James, R. G. & Bransby, P. L. (1970). Experimental and theoretical investigations of a passive earth pressure problem. *Géotechnique* **20**, No. 1, 17–37.

Janbu, N. & Senneset, K. (1974). Effective stress interpretation of in situ static penetration tests. *Proc. European Symp. Penetration Testing, Stockholm* **2.2**, 181–193.

Ladanyi, B. (1963). Evaluation of pressuremeter tests in granular soils. *2nd Panamerican Conf. Soil Mech. Fdn Engng* **1**, 3–20.

Laier, J. E., Schmertmann, J. H. & Schaub, J. H. (1975). Effect of finite pressuremeter length in dry sand. *Proc. Am. Soc. Civ. Engrs Specialty Conf. In Situ Measurement of Soil Properties*, Raleigh, North Carolina **I**, 241–259.

Meyerhof, G. G. (1974). General report on penetration testing outside Europe. *Proc. European Symp. Penetration Testing, Stockholm* **2.1**, 40–48.

Rowe, P. W. (1962). The stress–dilatancy relation for static equilibrium of an assembly of particles in contact. *Proc. Roy. Soc. A* **269**, 500–527.

Rowe, P. W. (1971). Theoretical meaning and observed values of deformation parameters for soil. *Proc. Roscoe Mem. Symp.*, pp. 143–194.

Schmertmann, J. H. (1975). Measurement of in situ shear strength. State-of-the-art report. *Am. Soc. Civ. Engrs Specialty Conf. In Situ Measurement of Soil Properties*, Raleigh, North Carolina **II**, 57–138.

Soil Mechanics Ltd (1971). *Wash Water Storage Scheme: Site Investigation.* Volume III. Laboratory Testing Report No. 5842/3.

Stroud, M. A. (1971). *Sand at low stress levels in the simple shear apparatus.* PhD thesis, University of Cambridge.

Tavenas, F. & La Rochelle, P. (1972). Accuracy of relative density measurements. *Géotechnique* **22**, No. 4, 549–562.

Terzaghi, K. & Peck, R. B. (1948). *Soil mechanics in engineering practice.* New York: Wiley.

Vesić, A. S. (1972). Expansion of cavities in infinite soil mass. *Jnl Soil Mech. Fdn Div. Am. Soc. Civ. Engrs* **98**, SM3, 265–290.

Wimpey Laboratories Ltd (1973). *Site investigation report to the Water Resources Board.* Field Work Volume I. Laboratory Reference No. S/9007.

Windle, D. & Wroth, C. P. (1977). *In situ measurement of the properties of stiff clays with self-boring instruments.* Paper submitted to 9th Int. Conf. Soil Mech. Fdn Engng, Tokyo.