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INTRODUCTION

High precision measurements of the deformation of stiff clay in the laboratory have shown that the response of these soils is non-linear from very small strains. This non-linear response is a reflection of the occurrence of plastic irrecoverable strains in the soil, and is an indication that the soil cannot realistically be described as a linear elastic material, as might traditionally have been assumed. Where a prototype problem involves only monotonic loading of the soil it may be feasible to interpret this inelastic non-linearity in terms of a non-linear elastic model, but for situations where the loading direction may vary during the construction process a description in terms of an elastic-plastic model is more appropriate. Prediction of performance of a geotechnical structure requires an integration of incremental movements within the soil, and this can be achieved most conveniently if the soil model describes the variation of the incremental stiffness of the soil as a function of deformation. The interpretation of variation of stiffness in pressuremeter tests will be presented in terms of elastic moduli which vary with strain level from the start of loading, even though the behaviour of the soil is not strictly elastic.

THEORY

Tangent, secant and pressuremeter moduli

The non-linear stress–strain response of the soil is described in terms of a tangential shear stiffness G_t which varies as a function of strain. In undrained triaxial compression tests the tangent shear modulus (Fig. 1(a)) is

$$3G_t = dq/d\varepsilon \quad (1)$$

where q is deviator stress $(\sigma_a - \sigma_r)$ and ε is triaxial shear strain $2(\varepsilon_a - \varepsilon_r)/3$.

The progress of non-linear deformation can be described less conveniently in terms of a varying secant modulus G_s

$$3G_s = \Delta q/\varepsilon \quad (2)$$

where the changes Δq and ε may have been measured either from the start of the test (Fig. 1(a)) or from the start of an unloading or reloading process (Fig. 1(b)). The tangent and secant shear moduli can be linked through a differential equation

$$G_t = G_s + \varepsilon\, dG_s/d\varepsilon \quad (3)$$

In the pressuremeter test, a pressuremeter shear modulus G_p is typically calculated from the slope of the chord of a cycle of unloading and reloading (Fig. 1(c)).

$$2G_p = \Delta p/\varepsilon_c \quad (4)$$

where Δp is the change in pressuremeter cavity pressure and ε_c is the change in cavity strain (equal to half the shear strain in the wall of the cavity) over which the modulus is being calculated—these changes being measured, for example, from the start of an unloading or a reloading process during a pressuremeter test. However, the subtangent construction, which is a small strain approximation to the finite deformation cavity expansion analysis presented by Palmer (1972), reveals that

$$dp/d\varepsilon_c = \Delta\tau/\varepsilon_c = 2G_s \quad (5)$$

where $\Delta\tau$ is the change in shear stress in the wall of the cavity. This result requires only that all elements of the soil should be experiencing the same constant volume plane strain stress–strain relationship. In the context of an unloading–reloading cycle this implies that the previous loading should not have affected the behaviour to be expected of the soil elements immediately following such a reversal of strain path. From equation (5), therefore, the current slope of the cavity pressure–cavity strain relationship gives a direct indication of the current value of the secant shear modulus. Combination of equations (4) and (5) then shows that

$$G_s = G_p + \varepsilon_c\, dG_p/d\varepsilon_c \quad (6)$$

The conclusion is that although the modulus G_p looks like a secant modulus, it does not on its own give a direct indication of the secant modulus of the soil. Equally, equation (5) shows that the tangent to the cavity pressure–cavity strain relationship does not in fact lead to the

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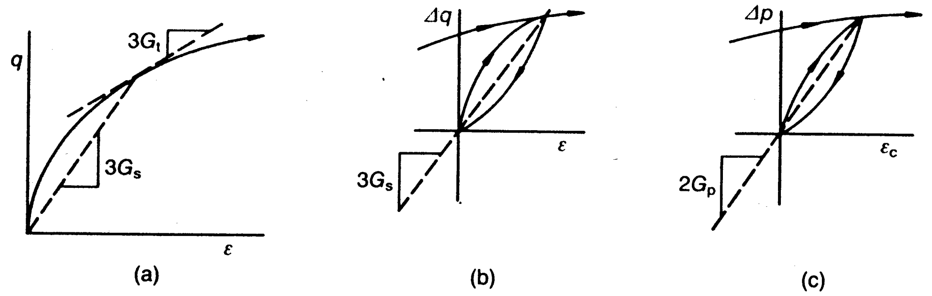

Fig. 1. (a) Tangent modulus G_t and secant modulus G_s in undrained triaxial compression test; (b) secant modulus G_s in unload–reload cycle of undrained triaxial compression test; (c) pressuremeter modulus G_p deduced from unload–reload cycle of undrained pressuremeter test

tangent stiffness of the soil, but is directly linked to a secant modulus. The conversion from secant to tangent modulus requires a differential equation such as equation (3).

Variation of moduli with strain

In order to show the effect of equations (3) and (6) on values of moduli it is necessary to assume some form for the variation of shear modulus with strain. Jardine, Potts, Fourie & Burland (1986) propose an expression for the variation of secant shear stiffness with strain of the form

$$G_s = A + B \cos \{\alpha[\ln (\varepsilon/C)]^{\gamma}\} \tag{7}$$

Since this expression is being used purely for illustrative purposes, there is some mathematical convenience to be obtained (at the expense of some loss of flexibility in fitting experimental data) by assuming that $\gamma = 1$ so that the modulus variation is controlled by the four parameters A, B, C and α

$$G_s = A + B \cos \{\alpha \ln (\varepsilon/C)\} \tag{8}$$

The nature of this expression can best be seen by plotting $(G_s - A)/B$ as a function of ε/C for different values of α (dotted curves in Fig. 2). The parameters A and B simply control the magnitudes of the moduli, while C provides a reference strain below which the modulus is not given by equation (8) but is assumed to have the constant value $(A + B)$. Thus C is treated as the threshold between elastic and inelastic response.

Equation (3) can be used to deduce an expression for the corresponding variation of tangent modulus with strain

$$G_t = A + B\{\cos [\alpha \ln (\varepsilon/C)] - \alpha \sin [\alpha \ln (\varepsilon/C)]\} \tag{9}$$

The nature of this expression is shown by the solid lines in Fig. 2, once again in terms of $(G_t - A)/B$ as a function of ε/C. Tangent modulus falls more steeply than secant modulus.

The curves in Fig. 2 have been terminated where equation (9) reaches a minimum. (It might be assumed for modelling purposes that the tangent stiffness remained constant at this low value for all higher strains. However, the nature of equations (3) and (6) implies that the secant stiffness G_s and the pressuremeter stiffness G_p will continue to fall.) Equation (9) can be written

$$G_t = A + B\sqrt{(1 + \alpha^2)} \cos [\alpha \ln (\varepsilon/C) + \theta] \tag{10}$$

where

$$\cos \theta = 1/\sqrt{(1 + \alpha^2)} \tag{11}$$

It is then very simple to see how the expression for tangent modulus can be fitted to experimental data.

$$\text{Initial modulus} = A + B \tag{12}$$

$$\text{Minimum modulus} = A - B\sqrt{(1 + \alpha^2)} \tag{13}$$

implying that

$$\text{Range of modulus values} = B[1 + \sqrt{(1 + \alpha^2)}] \tag{14}$$

The variation of this factor with α is shown in Fig. 3. The range of strain over which the modulus falls from its initial value to the minimum value is also controlled by α, and can be deduced from equation (10) to be from C to $C \exp [(\pi - \theta)/\alpha]$. The variation of this factor with α is shown in Fig. 4.

Equation (6) can be used with equation (8) to generate the relationship between pressuremeter modulus and strain

$$G_p = A + [B/\sqrt{(1 + \alpha^2)}] \times \{\cos [\alpha \ln (\varepsilon/C) - \theta] + \alpha/(\varepsilon/C)\} \tag{15}$$

This expression is plotted as the chain dotted curves in Fig. 2, in terms of $(G_p - A)/B$ as a function of ε/C for different values of α. As could be deduced from the form of the differential equations, this modulus is expected to fall much more

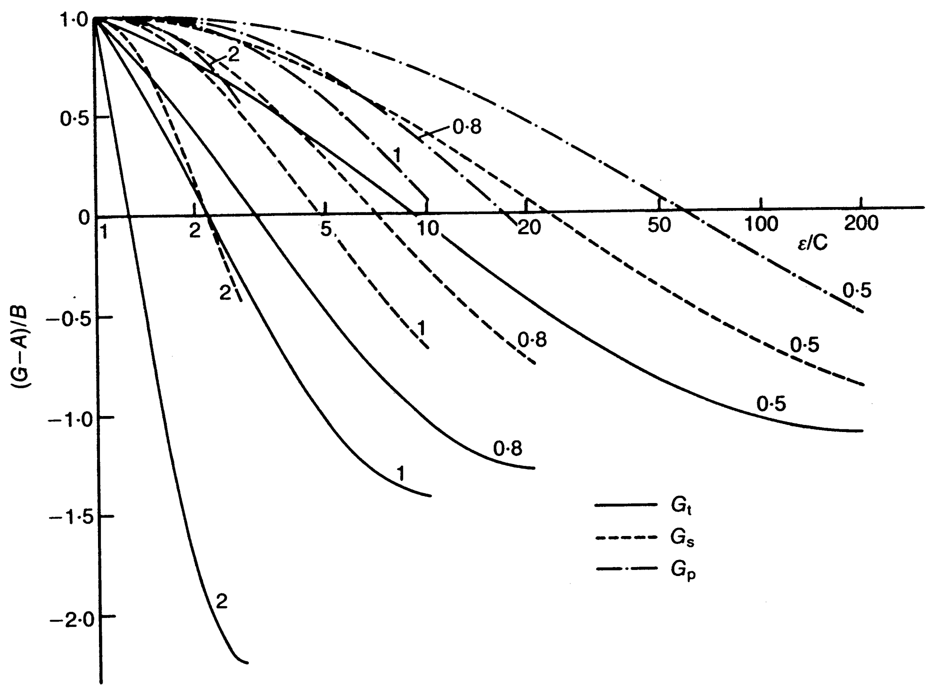

Fig. 2. Variation of dimensionless tangent stiffness (solid curves), secant stiffness (dotted curves) and pressuremeter stiffness (chain dotted curves) with normalized strain ε/C: numbers against curves indicate values of α

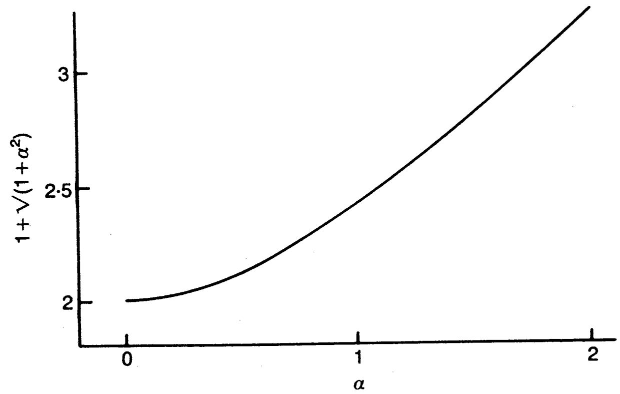

Fig. 3. Variation with α of factor $[1 + \sqrt{(1 + \alpha^2)}]$ which indicates range of values of modulus

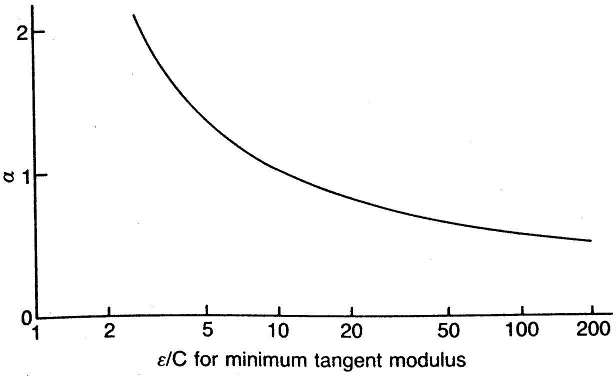

Fig. 4. Variation with α of normalized strain range ε/C over which tangent modulus falls from its initial value to its minimum value

slowly with strain. In writing equation (15) the distinction between triaxial shear strain ε and pressuremeter cavity strain ε_c has been temporarily blurred. For identical levels of octahedral shear strain the link between them is

$$\varepsilon = (2/\sqrt{3})\varepsilon_c \tag{16}$$

which is sufficiently close to unity to be neglected in the present analysis and, in any plotting of strains on a logarithmic scale, merely implies a small sideways shift of all the curves.

DISCUSSION

Attempts to fit pressuremeter and triaxial test data of small strain stiffness variation into the same picture run into the basic problem that the principal stiffness variations that are seen in the laboratory occur at extremely low strains, probably lower than the present resolution of pressuremeter testing equipment. There are further problems associated with the pressuremeter test itself.

Interpretation assumes that the expansion of a pressuremeter in clay is a truly undrained process. Excess pore pressures are generated during the process of cavity expansion and, since the cavity is not actually infinitely long, these will tend to dissipate vertically as well as radially. Many clays have a viscous element of response which is specifically excluded in the analysis leading to equation (5). The strain rate imposed by the pressuremeter varies inversely with radius, which implies both that the stress–strain relationship that the soil elements are following will not be the same at all radii, and also that relaxation or creep effects will tend to occur when there is a change in the process driving the deformation—for example, a change from loading to unloading or from unloading to reloading.

The net result is that it may be difficult to identify precisely the start of an unloading or reloading process, and that these effects may tend to obscure the small strain behaviour of the soil that is being sought. Superimposition of continuing outward creep deformation and inward unloading can lead to falsely high or even negative apparent initial unloading moduli.

Notwithstanding these possible difficulties in application it is the object of this Note to show that as soon as non-linear effects are to be studied, care is necessary in order to ensure that parameters that are being compared from pressuremeter and triaxial tests are indeed truly comparable. There is a more general moral, which is revealed also in the numerical analyses described by Jardine *et al.* (1986). Non-linearity of the form being considered here (reduction of stiffness with strain) leads to very different distributions of stress or strain around a test device, or around a geotechnical structure, from those which emerge from constant modulus elastic analyses. Treatment of a non-linear problem as a quasi-linear problem will give a completely false impression.

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