

AN ANALYTICAL SOLUTION FOR THE
CONSOLIDATION AROUND A DRIVEN
PILE

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SUMMARY

Field studies have shown that the driving of a displacement pile into cohesive soil generates large excess pore pressures in the vicinity of the pile. These pore pressures are often larger than the effective overburden pressure and facilitate the installation of the pile. The subsequent increase in bearing capacity of the pile is largely controlled by the dissipation of the excess pore pressures and a consequent increase in the effective stresses acting on the pile. The paper presents a closed form analytical solution for the radial consolidation of the soil around a driven pile, assuming that the soil skeleton deforms elastically. This assumption is examined in the light of the predicted effective stress changes in the soil and is shown to lead to a realistic model for the decay of pore pressure near the pile with time after driving. In addition to giving estimates of the time needed for a driven pile to achieve its maximum strength, the solution may also be used in the analysis of pressuremeter tests to provide in situ measurements of the coefficient of consolidation of the soil.

An Analytical Solution for the Consolidation Around a Driven Pile

by M.F.Randolph and C.P.Wroth

1. Introduction

When displacement piles are driven into a cohesive soil, large excess pore pressures are generated close to the pile. The high pore pressures down the pile shaft reduce the effective stresses and thus facilitate the process of driving the pile to the required penetration. Once installation is complete, the pore pressures will gradually dissipate, allowing the soil around the pile to consolidate. During consolidation, the soil increases in strength and the bearing capacity of the pile will increase accordingly. Seed and Reese (1955) report tests on a pile of radius 0.076 m driven into soft clay, which showed a sixfold increase in bearing capacity over a period of thirty days.

Relatively few accurate field data are available on the distribution of the excess pore pressures around a driven pile. However, the field measurements which have been made (e.g. Bjerrum and Johannessen, 1961; Lo and Stermac, 1965; Koizumi and Ito, 1967) show that the major pore pressure gradients are radial. Thus it will be assumed that consolidation takes place primarily by pore water flow radially outwards from the pile. The soil particles will move radially inwards towards the pile under conditions of plane strain and axial symmetry. Since the soil is moving back towards the pile (having been originally displaced outwards during driving) most of the soil will go through a process of unloading in shear. Thus in the first instance, it will be assumed that the soil skeleton will deform elastically during consolidation. The validity of this assumption is discussed at the end of the paper in the light of the predicted stress changes in the soil. The pile will be assumed to be rigid and impermeable.

At the end of driving, before consolidation has started, the excess pore pressure distribution will be of the form shown in Figure 1. For $r_0 \leq r \leq R$ (where r_0 is the radius of the pile), the excess pore pressure $u_0(r)$ is non-zero, while for $r > R$, u_0 is equal to zero. The solution for consolidation will be developed

for any, general, distribution of excess pore pressure $u_0(r)$. In the latter half of the paper, a particular distribution of $u_0(r)$, based on analytical considerations and the results of field measurements, will be considered in detail. Typical distributions of excess pore pressure around a driven pile at various times after driving, will be shown.

2. Derivation of Equations Governing Consolidation

The basic equations which govern the process of consolidation will be outlined below. Since it is assumed that the soil skeleton will deform elastically, only increments of stress (from the stress state immediately after driving) will be considered. Similarly, the symbol u will be used to denote the excess pore pressure (above hydrostatic) at a given time and radius. Conditions of axial symmetry and plane strain imply that the soil particles have only one degree of freedom - that of radial movement. The symbol ξ will be used to denote the outward radial movement of the soil particles. Compressive stresses and strains will be taken as positive.

(i) Stress-strain behaviour of soil skeleton

The plane strain version of Hooke's laws is

$$\epsilon_r = - \frac{\partial \xi}{\partial r} = \frac{1}{2G} [(1-\nu) \delta \sigma'_r - \nu \delta \sigma'_\theta] \quad (1)$$

$$\epsilon_\theta = - \frac{\xi}{r} = \frac{1}{2G} [-\nu \delta \sigma'_r + (1-\nu) \delta \sigma'_\theta] \quad (2)$$

$$\epsilon_z = 0 \quad (3)$$

These equations may be inverted to give the stress increments explicitly, i.e.

$$\delta \sigma'_r = \frac{-2G}{(1-2\nu)} [(1-\nu) \frac{\partial \xi}{\partial r} + \nu \frac{\xi}{r}] \quad (4)$$

$$\delta \sigma'_\theta = \frac{-2G}{(1-2\nu)} [\nu \frac{\partial \xi}{\partial r} + (1-\nu) \frac{\xi}{r}] \quad (5)$$

$$\delta\sigma'_z = v(\delta\sigma'_r + \delta\sigma'_\theta) = \frac{-2G}{(1-2v)} \left[\frac{\partial \xi}{\partial r} + \frac{\xi}{r} \right] \quad (6)$$

In these equations, G is the shear modulus of the soil, and v is the Poisson's ratio.

(ii) Radial equilibrium

The equation of radial equilibrium in terms of total stresses is

$$\frac{\partial}{\partial r} (r \delta\sigma_r) - \delta\sigma_\theta = 0 \quad (7)$$

The total stress increments may be written in terms of the effective stress increments and the change in excess pore pressure as

$$\delta\sigma_r = \delta\sigma'_r + \delta u = \delta\sigma'_r + u - u_0 \quad (8)$$

$$\delta\sigma_\theta = \delta\sigma'_\theta + \delta u = \delta\sigma'_\theta + u - u_0 \quad (9)$$

Substitution of these equations into equation (7) yields

$$\frac{\partial u}{\partial r} = \frac{du_0}{dr} - \frac{\partial}{\partial r} (\delta\sigma'_r) + \frac{\delta\sigma'_\theta - \delta\sigma'_r}{r} \quad (10)$$

Note that the derivative of u_0 may be written as a total derivative since u_0 is a function of radius only and not of time. Finally, substitution of equations (4) and (5) leads to

$$\frac{\partial u}{\partial r} = \frac{du_0}{dr} + G^* \frac{\partial}{\partial r} \left[\frac{1}{r} \frac{\partial}{\partial r} (r \xi) \right] \quad (11)$$

where $G^* = \frac{2G(1-v)}{(1-2v)}$.

(iii) Flow of pore water and continuity of volume strain rate

The (artificial) velocity of the pore water relative to the soil particles is given by Darcy's law in terms of the pressure gradient as

$$v = \frac{-k}{\gamma_w} \frac{\partial u}{\partial r} \quad (12)$$

where k is the permeability of the soil and γ_w is the unit weight of water.

For continuity, the rate of volumetric strain must be related to the flow of pore water into and out of any region by

$$\frac{\partial}{\partial t} (\epsilon_r + \epsilon_\theta + \epsilon_z) = \frac{1}{r} \frac{\partial}{\partial r} (r v) \quad (13)$$

Equations (12) and (13) may be combined, making use of the expressions for the strains, to give

$$\frac{k}{\gamma_w} \left[\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial u}{\partial r} \right) \right] = \frac{\partial}{\partial t} \left[\frac{1}{r} \frac{\partial}{\partial r} (r \xi) \right] \quad (14)$$

Changing the order of differentiation, and integrating with respect to the radius r , yields

$$\frac{k}{\gamma_w} \frac{\partial u}{\partial r} = \frac{\partial \xi}{\partial t} + \frac{f(t)}{r} \quad (15)$$

where $f(t)$ is a constant of integration (and thus may be a function of time t).

The final governing equation may be formed by eliminating either u or ξ between equations (11) and (15). Thus, eliminating the radial movement ξ leads to

$$\frac{\partial u}{\partial t} = c \left[\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial u}{\partial r} \right) \right] + g(t) = c \nabla^2 u + g(t) \quad (16)$$

where $c = \frac{k}{\gamma_w}$ $G^* = \frac{k}{\gamma_w} \frac{2G(1-\nu)}{(1-2\nu)}$.

A constant of integration $g(t)$ has been introduced, the value of which will depend on the boundary conditions. It is interesting to note that, apart from the constant of integration, equation (16) is identical to Terzaghi's one-dimensional consolidation equation. The coefficient of consolidation c is the same as that for one-dimensional consolidation c_{vc} , since $c_{vc} = \frac{k}{\gamma_w} \cdot \frac{1}{m_v}$ and m_v (the volume compressibility) is equal to $\frac{1}{G^*}$. This result might have been foreseen considering that in both cases there is only one degree of freedom and, at large radii, the geometry of the radial

consolidation becomes similar to that of one-dimensional consolidation.

The governing equation in ξ may be obtained by eliminating u to give

$$\frac{\partial \xi}{\partial t} = \frac{k}{\gamma_w} \frac{du}{dr} + c \left\{ \frac{\partial}{\partial r} \left[\frac{1}{r} \frac{\partial}{\partial r} (r \xi) \right] \right\} - \frac{f(t)}{r} \quad (17)$$

3. Boundary Conditions

The boundary conditions for the consolidation of soil around a rigid, impermeable pile are

- (i) $\xi = 0$ at $t = 0$ for $r \geq r_0$
- (ii) $\xi = 0$ at $r = r_0$ for $t \geq 0$
- (iii) $\xi \rightarrow 0$ as $r \rightarrow \infty$ for $t \geq 0$
- (iv) $u = u_0$ at $t = 0$ for $r \geq r_0$
- (v) $u \rightarrow 0$ as $r \rightarrow \infty$ for $t \geq 0$
- (vi) $u \rightarrow 0$ as $t \rightarrow \infty$ for $r \geq r_0$
- (vii) $\frac{\partial u}{\partial r} = 0$ at $r = r_0$ for $t > 0$.

Some simplification of the governing equations is now possible since conditions (ii) and (vii) imply that the constant of integration $f(t)$ is zero in equations (15) and (17). Also, condition (v) implies that $g(t)$ is zero in equation (16).

Solutions to equations of the form of (16) and (17) have been derived by Carslaw and Jaeger (1959), but these are mathematically intractable and a common method is to solve the equations numerically using a finite difference approach (Banerjee, 1970). It is well known that Bessel functions of zero order are solutions of equation (16), but these do not lend themselves to satisfying the boundary conditions as $r \rightarrow \infty$. However, a slight modification to these boundary conditions enables a solution to be found in terms of such functions. Considering Figure 1, initially the excess pore pressure will be zero for $r \geq R$. As consolidation progresses, the radius at which the excess pore pressure is zero, will increase.

Finally at large values of t , the excess pore pressure will tend to zero at all radii. There will be some radius, which may be designated r^* , at which the excess pore pressure is never more than negligibly small. Thus r^* is some radius ($> R$) at which the excess pore pressures, by the time they reach there, are small and maybe ignored for $r \geq r^*$.

The problem to be solved is now that of soil surrounding a pile where the soil permeability is finite ($= k$) for $r_0 \leq r \leq r^*$ and infinite (i.e. $u = 0$) for $r > r^*$. The new boundary condition (v) is $u = 0$ at $r = r^*$. The value chosen for r^* may be varied to find how sensitive the solution is to the choice. Typically, it is to be expected that r^* will be of the order of 5 or 10 times R , although the solution for times close to $t = 0$ may be obtained with much smaller values of r^* .

4. General Solution

The general solution to equations (16) and (17) may be obtained by separating the variables (Carslaw and Jaeger, 1959) to give, for a separation constant of $-\alpha^2$,

$$u = B e^{-\alpha^2 t} [J_0(\lambda r) + \mu Y_0(\lambda r)] \quad (18)$$

$$\xi = \frac{B}{G^* \lambda} e^{-\alpha^2 t} [J_1(\lambda r) + \mu Y_1(\lambda r)] + h(r) . \quad (19)$$

The functions J_0, Y_0, J_1, Y_1 are Bessel functions of zero order and first order, J_i being Bessel functions of the first kind, and Y_i being Bessel functions of the second kind. The linear combination of $J_i(\lambda r) + \mu Y_i(\lambda r)$ is a cylinder function of the i^{th} order and will be written as $G_i(\lambda r)$. Further information on the properties of cylinder functions may be obtained from McLachlan (1934).

Boundary conditions (ii) and (vii) imply that

$$G_1(\lambda r_0) = J_1(\lambda r_0) + \mu Y_1(\lambda r_0) = 0 . \quad (20)$$

Also, since $u = 0$ for $r \geq r^*$, the constant B must be zero for the part of the solution for $r > r^*$. In order that $u = 0$ at $r = r^*$

$$G_0(\lambda r^*) = J_0(\lambda r^*) + \mu Y_0(\lambda r^*) = 0 . \quad (21)$$

Equations (20) and (21) enable the cylinder functions to be defined. Since the Bessel functions are periodic, there will be an infinite number of values of λ and μ which satisfy these equations. The full expressions for u and ξ will involve a summation of all the possible solutions, i.e.

$$\left. \begin{aligned} u &= \sum_{n=1}^{\infty} B_n e^{-\alpha_n^2 t} G_0(\lambda_n r) & r_0 \leq r \leq r^* \\ u &= 0 & r > r^* \end{aligned} \right\} \quad (22)$$

$$\left. \begin{aligned} \xi &= \frac{1}{G^*} \sum_{n=1}^{\infty} \frac{B_n}{\lambda_n} e^{-\alpha_n^2 t} G_1(\lambda_n r) + h(r) & r_0 \leq r \leq r^* \\ \xi &= \frac{F(t)}{r} & r > r^* \end{aligned} \right\} \quad (23)$$

The function $F(t)$ may be found by ensuring continuity of displacement at $r = r^*$.

In order to evaluate the coefficients B_n , techniques of Fourier-Bessel analysis are needed (see McLachlan, 1934).

At $t = 0$, the excess pore pressures must be given by

$$u_0 = \sum_{n=1}^{\infty} B_n G_0(\lambda_n r) \quad (24)$$

Multiplying both sides of this equation by $r G_0(\lambda_n r)$ and integrating between r_0 and r^* yields, using the orthogonal properties of Bessel functions,

$$\int_{r_0}^{r^*} u_0 r G_0(\lambda_n r) dr = \frac{B_n}{2} [r^{*2} G_1^2(\lambda_n r^*) - r_0^2 G_0^2(\lambda_n r_0)] \quad (25)$$

Thus, provided the left hand side of this equation may be integrated, this leads to the formal solution of the consolidation problem.

5. Initial Excess Pore Pressure Distribution

Sophisticated piezometers and instrumentation techniques are needed to enable pore pressures to be measured in low permeability materials such as clay. Attempts to measure the pore pressure distribution in the vicinity of a driven pile have usually produced scattered and inconclusive results. Figure 2 shows some typical data collected from Bjerrum and Johannessen (1961), Lo and Stermac (1965), and Koizumi and Ito (1967). It may be seen that excess pore pressures close to or greater than the effective overburden stress are set up close to the pile. Although there is much scatter, the magnitude of the excess pore pressures appears to decrease approximately linearly with the logarithm of the radius from the pile axis, as indicated by the broken lines.

When a pile is driven into clay, some surface heave occurs at small amounts of penetration. At large penetrations (greater than about ten radii) little surface heave is noticed and the soil must move predominantly outwards. This general form of soil movement has led to the modelling of the installation process as an expansion of a cylindrical cavity from zero radius to the radius of the pile (Soderberg, 1965; Banerjee, 1970). Measurements of the soil movement around piles driven under laboratory conditions are in general accordance with such a model (Clark and Meyerhof, 1972; Roy, Michaud, Tavenas, Leroueil and La Rochelle, 1975).

It is possible to make use of analyses developed for the interpretation of pressuremeter tests in order to estimate the stress changes which occur when a cylindrical cavity is expanded. In particular, if the soil is modelled as an ideal elastic, perfectly plastic material with a shear modulus G and undrained shear strength c_u , then the expressions for the stresses around an expanded cavity are given by Hill (1950) and Gibson and Anderson (1963). For a cavity expanded from zero radius to a radius of r_0 (the radius of the pile), the radial and circumferential stress changes within the plastic zone ($r_0 \leq r \leq R$) are given by

$$\delta\sigma_r = c_u [1 + \ln(G/c_u) - 2 \ln(r/r_0)] \quad (26)$$

$$\delta\sigma_\theta = c_u [-1 + \ln(G/c_u) - 2 \ln(r/r_0)] . \quad (27)$$

The width of the plastic zone is given by

$$R = r_o [G/c_u]^{\frac{1}{2}} . \quad (28)$$

The excess pore pressures generated may be estimated by assuming that, under undrained conditions, the mean effective stress remains constant. Thus the excess pore pressure is equal to the change in mean total stress, giving

$$u = \frac{1}{3}(\delta\sigma_r + \delta\sigma_\theta + \delta\sigma_z) = \frac{1}{2}(\delta\sigma_r + \delta\sigma_\theta) = \quad (29)$$

$$c_u [\ln(G/c_u) - 2 \ln(r_o/r)] .$$

Outside the plastic zone, the excess pore pressure will be zero since $\delta\sigma_r$ and $\delta\sigma_\theta$ are equal and opposite in size ($\delta\sigma_r = -\delta\sigma_\theta = c_u (\frac{R}{r})^2$). Thus the initial pore pressure distribution to be substituted into the consolidation solution, may be written

$$\left. \begin{aligned} u_o &= 2 c_u \ln(R/r) & r_o \leq r \leq R \\ u_o &= 0 & R < r \leq r^* \end{aligned} \right\} \quad (30)$$

where $R/r_o = (G/c_u)^{\frac{1}{2}}$.

For the case of a hollow pile, or where some allowance is made for vertical movement of the soil during driving of the pile, the increase in volume of the cylindrical cavity may be reduced by a factor β which, for a hollow pile, will be the ratio of net volume to gross volume of the pile. By considering the cavity as being expanded from an initial radius r_i to the pile radius r_o , where $\beta = (r_o^2 - r_i^2)/r_o^2$, the excess pore pressure distribution immediately after driving is given by equation (30) but with the radius of the plastic zone given by

$$(R/r_o) = (\beta G/c_u)^{\frac{1}{2}} . \quad (31)$$

It should be noted that the assumption of an elastic perfectly plastic soil model, although clearly an idealisation, may lead to reasonable predictions for the excess pore pressures generated provided suitable values for the secant shear modulus are chosen (see Ladanyi, 1973; Marsland and Randolph, 1977). Alternatively if the

results of pressuremeter tests for a particular soil are available then the total radial stress on the side of a pile driven into that soil may be estimated from the limit pressure needed to cause infinite expansion of the pressuremeter. The excess pore pressure generated at the pile shaft may then be estimated assuming that, as for the case for the elastic, perfectly plastic soil,

$$(u_o)_{r_o} = (\delta\sigma_r)_{r_o} - c_u \quad (32)$$

Similarly, the excess pore pressures may be assumed to decrease linearly with the logarithm of (r/r_o) with a slope of $2c_u$ (see for example, the broken lines in Figure 2).

6. Consolidation Solution for Logarithmic Variation of u_o

The initial excess pore pressure distribution given by equation (30) may be substituted into the general consolidation solution (equations (22), (23) and (25)). The function $h(r)$ in the expression for the radial movement ξ must be split into two parts, for $r_o \leq r \leq R$ and $R < r \leq r^*$, since it depends on u_o . The final solution is

$$\left. \begin{aligned} u &= \sum_{n=1}^{\infty} B_n e^{-\alpha_n^2 t} \mathcal{C}_0(\lambda_n r) & r_o \leq r \leq r^* \\ u &= 0 & r > r^* \end{aligned} \right\} \quad (33)$$

$$\left. \begin{aligned} \xi &= \frac{1}{G^*} \sum_{n=1}^{\infty} \frac{B_n}{\lambda_n} e^{-\alpha_n^2 t} \mathcal{C}_1(\lambda_n r) + \frac{c_u}{G^*} \left[r \ln\left(\frac{r}{R^*}\right) - \frac{r_o^2}{r} \ln\left(\frac{r_o}{R^*}\right) \right] & r_o \leq r \leq R \\ \xi &= \frac{1}{G^*} \sum_{n=1}^{\infty} \frac{B_n}{n} e^{-\alpha_n^2 t} \mathcal{C}_1(\lambda_n r) + \frac{c_u}{G^* r} \left[R^2 \ln\left(\frac{R}{R^*}\right) - r_o^2 \ln\left(\frac{r_o}{R^*}\right) \right] & R < r \leq r^* \\ \xi &= \frac{F(t)}{r} & r > r^* \end{aligned} \right\} \quad (34)$$

where R^* has been written for $R\sqrt{e}$, so that $\frac{1}{2} + \ln(R) = \ln(R^*)$.

The coefficients B_n may be established by performing the integration in equation (25) to give

$$B_n = \frac{4 c_u}{\lambda_n^2} \frac{[G_0(\lambda_n r_0) - G_0(\lambda_n R)]}{[r^{*2} G_1^2(\lambda_n r^*) - r_0^2 G_0^2(\lambda_n r_0)]} \quad (35)$$

Details of the computing algorithms used to evaluate the solution are given by Randolph (1977). The radius r^* was taken as between 5 and 10 times R , depending on the time allowed to elapse since driving. Fifty Bessel functions terms were found to give sufficient accuracy (checked by comparing u at time $t = 0$ with the exact expression for u_0).

7. Variation of Excess Pore Pressure During Consolidation

From equation (30) the initial excess pore pressure at the pile shaft is proportional to the undrained shear strength c_u ; it also depends on the ratio G/c_u . In presenting charts of variation of pore pressure with time, it seems sensible to normalise the pore pressures by the undrained shear strength (see Butterfield and Johnson, 1973). Figure 3 shows the variation of u/c_u at the pile-soil interface with time, for soils with different values of G/c_u . The time axis has been non-dimensionalised as $\tau = ct/r_0^2$. This dimensionless group was suggested by Soderberg (1965) and follows directly from the time constant $1/\alpha^2$ in the present solution. The quantity λr_0 is a zero of the cylinder function and thus, in effect, a mathematical constant, z . The time for a given amount of consolidation is proportional to $1/\alpha^2$ and thus (from equation (19)) to $r_0^2/(c z^2)$, where λ has been replaced by z/r_0 . Thus by replacing the time t by $\tau = ct/r_0^2$, the consolidation curves for a range of values of c and r_0 will all fall on one curve.

Figure 4 shows the consolidation process for a soil with $G/c_u = 50$, a typical value for soft clay. The excess pore pressure distribution with radius at different values of ct/r_0^2 is shown; it is clear that non-zero excess pore pressures quickly develop at radii greater than the initial plastic zone. Note that the

pore pressure close to the pile falls off very rapidly initially. This phenomenon has been reported by Seed and Reese (1955) and by Eide, Hutchinson and Landva (1961) who showed very rapid increases in bearing capacity of a driven pile within a short time after driving. Figure 5 shows a comparison between the theoretical decay of excess pore pressure predicted by the above solution and the measured bearing capacity of driven piles as a percentage of their long term bearing capacity. The time-scale has been normalised by dividing by the time taken for 90% of the consolidation to have occurred. The theoretical curve was obtained for a G/c_u ratio of 100 (estimated from the soil data of Seed and Reese (1955) and Eide et al (1961)); because of the manner in which the time-scale has been normalised, this curve is relatively insensitive to the value of G/c_u chosen.

8. Changes in Principal Stresses During Consolidation

In order to check the validity of the assumption that the soil skeleton deforms elastically during consolidation, the principal stress changes must be examined. In addition these stress changes are of great importance in estimating the bearing capacity of a driven pile at any given time after installation. The stress changes may be calculated by substituting the solution into equations (4), (5) and (6). The values at the pile shaft in terms of the change in pore pressure are

$$\delta\sigma'_r = -\delta u \quad (36)$$

$$\delta\sigma'_\theta = \frac{\nu}{1-\nu} (-\delta u) \quad (37)$$

$$\delta q = \delta\sigma'_r - \delta\sigma'_\theta = \frac{(1-2\nu)}{(1-\nu)} (-\delta u) \quad (38)$$

$$\delta p' = \frac{1}{3} (\delta\sigma'_r + \delta\sigma'_\theta + \delta\sigma'_z) = \frac{(1+\nu)}{3(1-\nu)} (-\delta u) \quad (39)$$

Two important points need to be made. Firstly, the predicted change in radial effective stress is equivalent to saying that the total radial stress change is zero. Thus the solution implies that the full limiting pressure needed to expand the cylindrical cavity will eventually act as effective stress against the pile. The

second point concerns the ratio of deviatoric stress increment δq , to mean effective stress increment $\delta p'$. This ratio is

$$\frac{\delta q}{\delta p'} = \frac{3(1-2\nu)}{(1+\nu)} \quad (40)$$

For reasonable values of Poisson's ratio ν , this ratio is approximately unity. This casts doubt on the validity of the assumption that the soil is unloading (decreasing q) and thus deforming elastically. If the deviatoric stress increment is examined elsewhere in the soil, it is in fact found to be negative at radii greater than about 2 or 3 times r_0 . Thus most of the soil is unloading regarding shear, while the mean stress is increasing.

The solution for the time dependence of the excess pore pressures will be accurate since the consolidation process is largely controlled by the flow of pore water through the large volume of soil at intermediate radii. The consolidation of soil around an expanded cylindrical cavity may be analysed with a variety of soil models using the finite element method. Preliminary results (Carter, Randolph and Wroth, 1978) assuming either an elastic, perfectly plastic or a work-hardening elastoplastic soil model, show that the calculated dissipation of pore pressures with time are relatively unaffected by the assumption of a linear soil model. However, the predicted stress changes are much more dependent on the type of soil model. In particular, the finite element analyses show a smaller predicted increase in the effective radial stress than that given by equation (36).

It is possible to measure the total and effective radial stress changes around an expanded cylindrical cavity, by conducting strain-controlled tests with a pressuremeter fitted with a pore pressure transducer. Such tests have been carried out by Clarke (1977) at Cambridge University. Clarke found that, as might be expected, there was a small decrease with time of the total radial stress around a pressuremeter held at a fixed expanded volume. By monitoring the decay of pore pressure with time, good estimates of the in-situ coefficient of consolidation were obtained, using curves similar to those in Figure 3.

9. Conclusions

An analytical solution has been presented for the consolidation of soil around a driven pile, based on radial flow of pore water. The soil skeleton has been assumed to deform elastically under plane strain conditions. The solution applicable to an impermeable rigid pile has been studied in detail. Good correlation has been shown between the predicted decay of pore pressure close to the pile and the increase in bearing capacity of driven piles. However, a more sophisticated soil model is needed if the predicted changes in effective stress close to the pile are to be used to calculate the final bearing capacity of a driven pile.

It is important to be able to estimate the time necessary for a pile to attain its full strength. Figure 3 shows that 90% consolidation around a solid driven pile occurs at times between $20 r_o^2/c$ and $60 r_o^2/c$, depending on the initial maximum excess pore pressure generated. Conversely, the solution may be used to obtain estimates of the in-situ coefficient of consolidation by studying the time dependence of the excess pore pressures around a driven pile, or around a pressuremeter expanded to a fixed volume.

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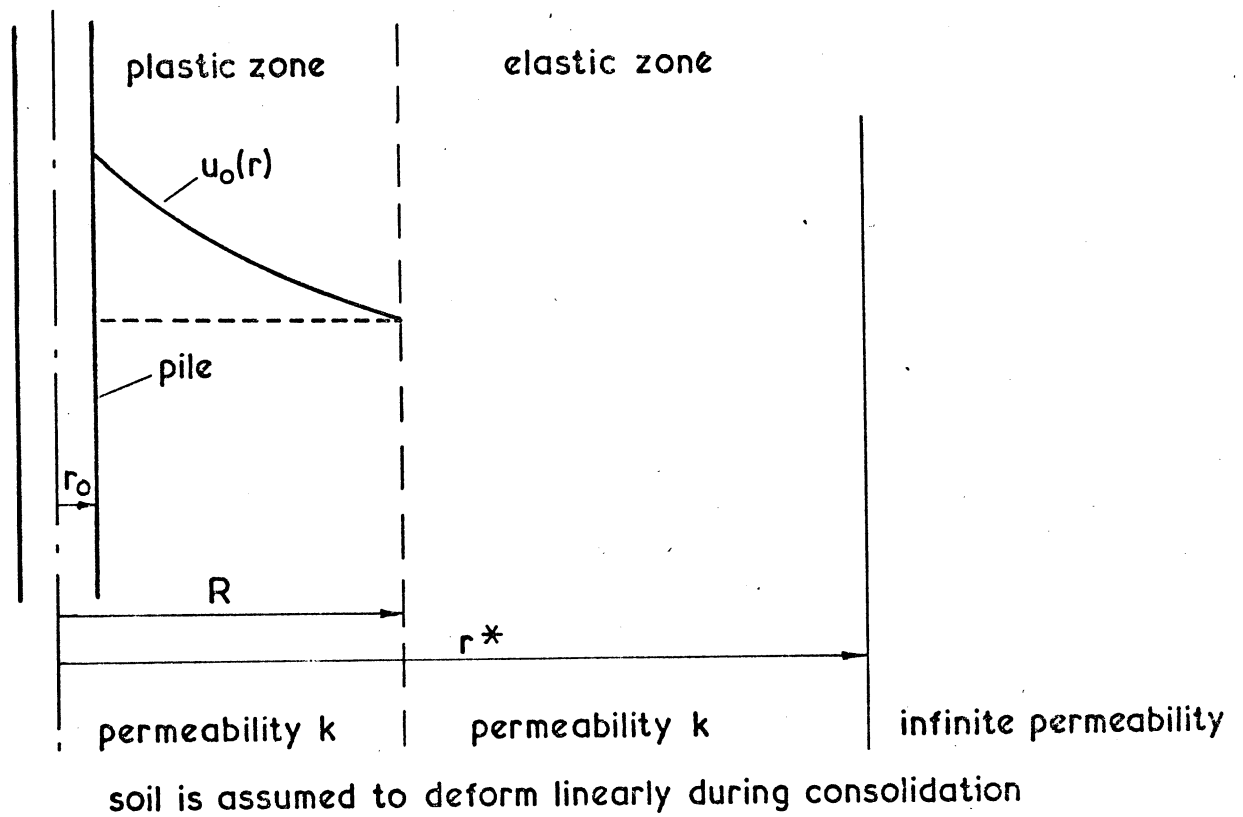


Figure 1 Diagram of soil around driven pile showing features of consolidation solution.

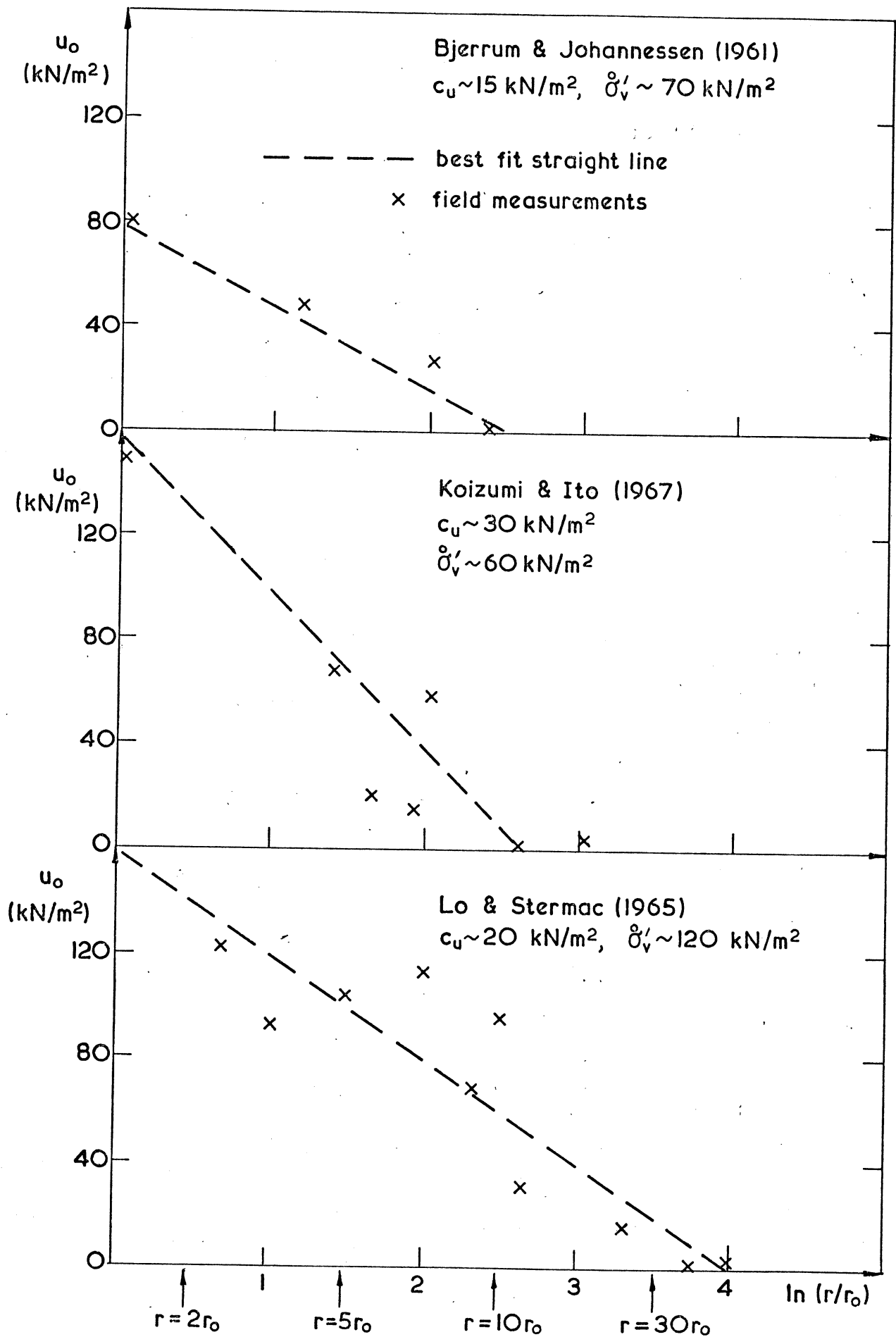


Figure 2 Field measurements of excess pore pressures resulting from pile driving.

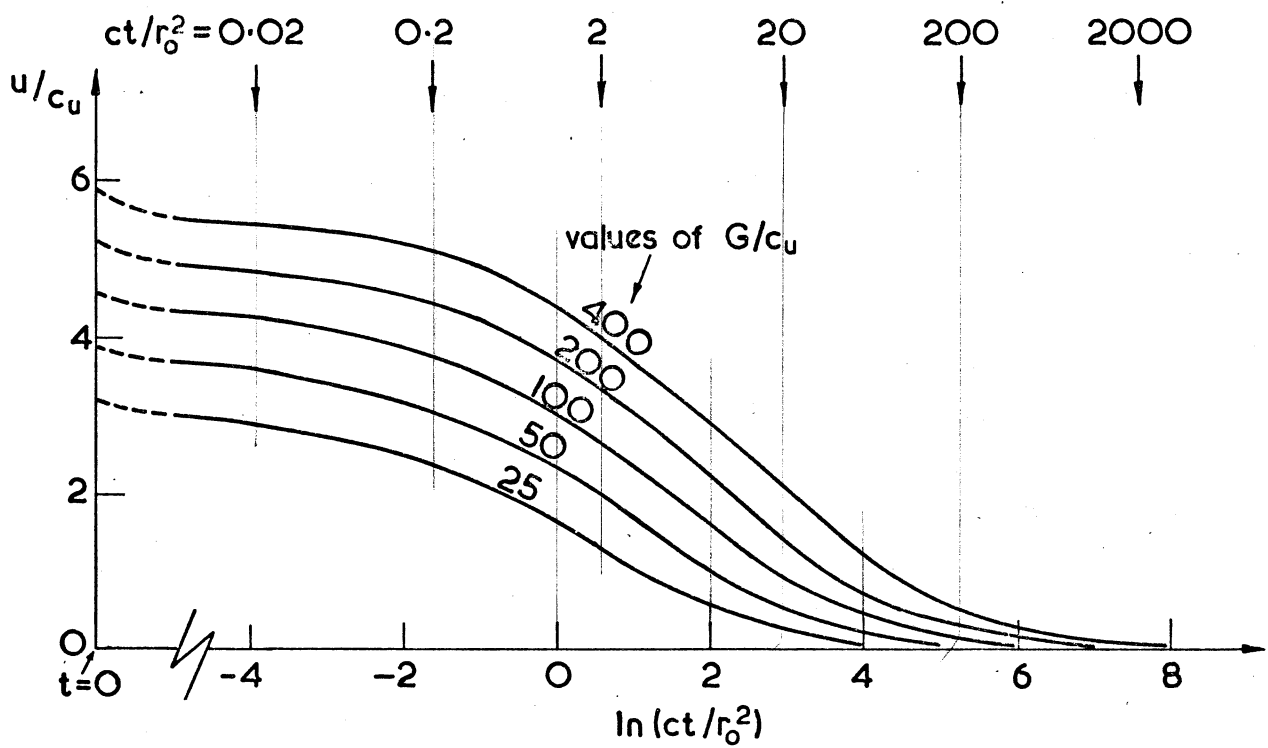


Figure 3 Variation of excess pore pressure at the pile face with time.

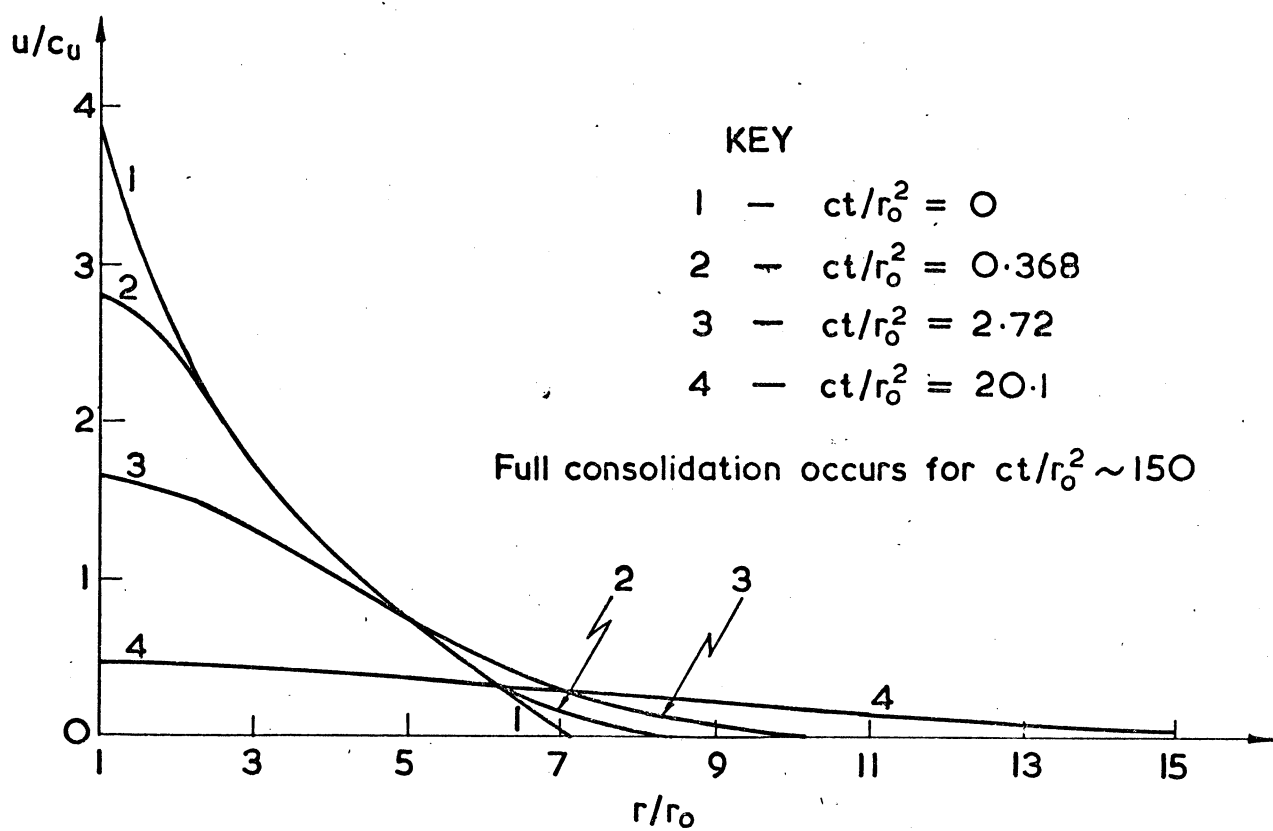


Figure 4 Variation of radial distribution of excess pore pressure with time after driving for soil with $G/c_u = 50$.

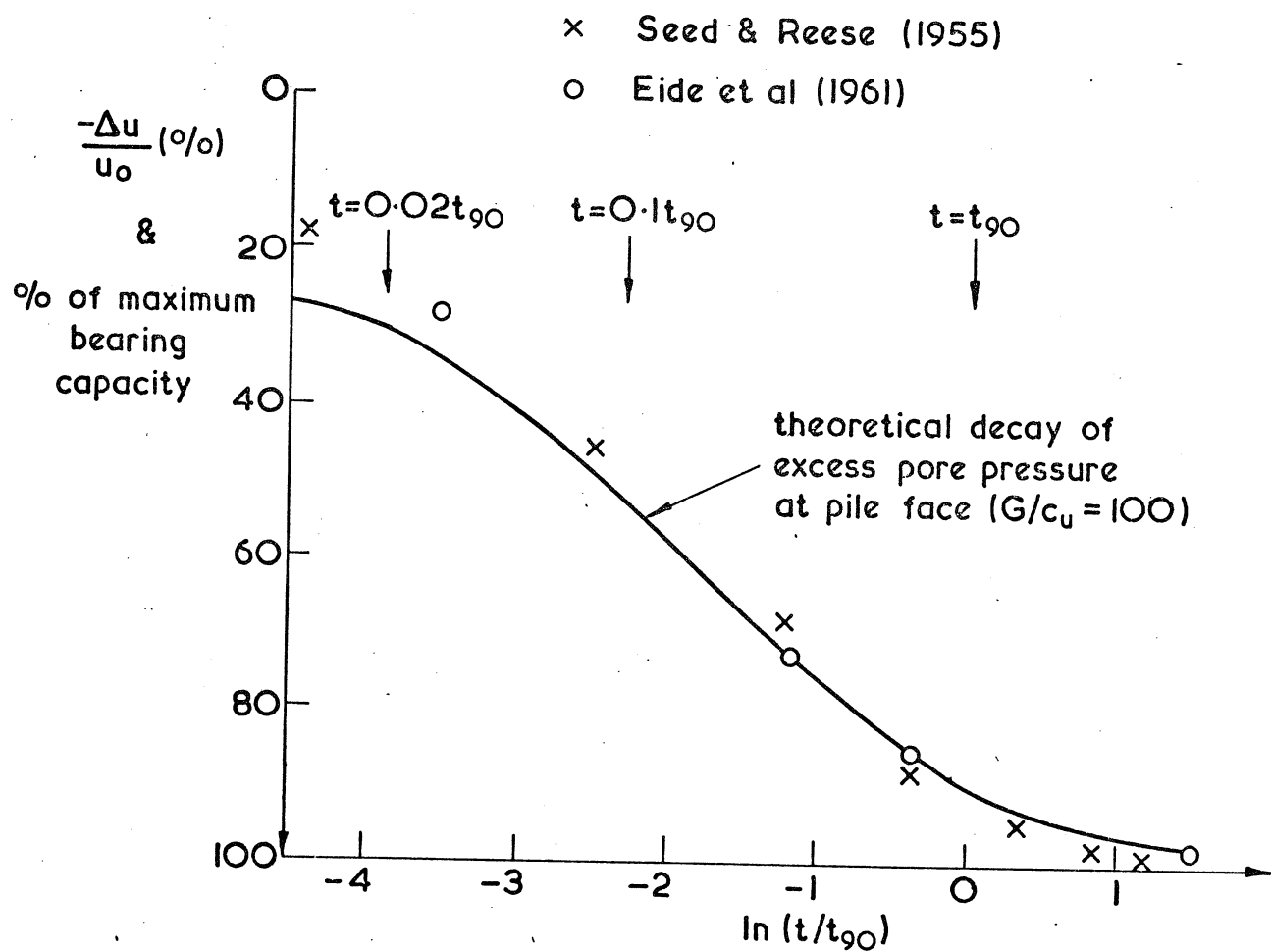


Figure 5 Comparison of variation of pile bearing capacity with time and theoretical decay of excess pore pressure.